

1)

- a. The model for a 2x3x5 factorial treatment structure with $n = 6$ replications and factors A and B random and C fixed is as follows:

$$y_{ijkm} = \mu + \alpha_i + \beta_j + \gamma_k + \alpha\beta_{ij} + \alpha\gamma_{ik} + \beta\gamma_{jk} + \alpha\beta\gamma_{ijk} + \epsilon_{ijkm}; i = 1, 2; j = 1, 2, 3; k = 1, \dots, 5;$$

$m = 1, \dots, 6$ where y_{ijkm} is the response of the m^{th} replication under level i factor A, level j factor B, level k factor C.

μ is the mean of all responses

α_i is the random effect of the i^{th} level of factor A

β_j is the random effect of the j^{th} level of factor B

γ_k is the fixed effect of the k^{th} level of factor C

$\alpha\beta_{ij}$ is the random interaction effect of the i^{th} level of factor A with the j^{th} level of factor B

$\alpha\gamma_{ik}$ is the random interaction effect of the i^{th} level of factor A with the k^{th} level of factor C

$\beta\gamma_{jk}$ is the random interaction effect of the j^{th} level of factor B with the k^{th} level of factor C

$\alpha\beta\gamma_{ijk}$ is the random interaction effect of the i^{th} level of factor A with the j^{th} level of factor B and the k^{th} level of factor C

ϵ_{ijkm} is the random error associated with the m^{th} replicate of levels i, j , and k of factors A, B, and C respectively

- b. The AOV table is shown below:

Source	df	SS	MS	Expected MS	Denom of F
A	1	SSA	SSA/1	$\sigma_\epsilon^2 + 6\sigma_{\alpha\beta\gamma}^2 + 30\sigma_{\alpha\beta}^2 + 18\sigma_{\alpha\gamma}^2 + 90\sigma_\alpha^2$	*
B	2	SSB	SSB/2	$\sigma_\epsilon^2 + 6\sigma_{\alpha\beta\gamma}^2 + 30\sigma_{\alpha\beta}^2 + 12\sigma_{\beta\gamma}^2 + 60\sigma_\beta^2$	*
C	4	SSC	SSC/4	$\sigma_\epsilon^2 + 6\sigma_{\alpha\beta\gamma}^2 + 18\sigma_{\alpha\gamma}^2 + 12\sigma_{\beta\gamma}^2 + 36\theta_C$	*
AB	2	SSAB	SSAB/2	$\sigma_\epsilon^2 + 6\sigma_{\alpha\beta\gamma}^2 + 30\sigma_{\alpha\beta}^2$	MSABC
AC	4	SSAC	SSAC/4	$\sigma_\epsilon^2 + 6\sigma_{\alpha\beta\gamma}^2 + 18\sigma_{\alpha\gamma}^2$	MSABC
BC	8	SSBC	SSBC/8	$\sigma_\epsilon^2 + 6\sigma_{\alpha\beta\gamma}^2 + 12\sigma_{\beta\gamma}^2$	MSABC
ABC	8	SSABC	SSABC/8	$\sigma_\epsilon^2 + 6\sigma_{\alpha\beta\gamma}^2$	MSE
Error	150	SSE	SSE/150	σ_ϵ^2	*
Total	179	SST			

*A MSAB + MSAC - MSABC

*B MSAB + MSBC - MSABC

*C MSAC + MSBC - MSABC

2)

17.33

- a. Fixed: Soil Type(So)—Because the specific soil types are selected by the researcher deterministically
 Random: Site (Si)—Because the sites are randomly chosen from a given soil type
- b. The factors are nested (Site within Soil). This is nested because only certain sites contain a given soil type.
- c. The AOV table is given below:

Source	df	Expected MS
So	2	$\sigma_{\epsilon}^2 + 5\sigma_{So(Si)}^2 + 25\theta_{So}$
Si(So)	12	$\sigma_{\epsilon}^2 + 5\sigma_{So(Si)}^2$
Error	60	σ_{ϵ}^2
Total	74	

3) [(i:k) (j)]:1

[{Batches are nested within Days} crossed with Lab] each has replication ...

4)

2 stage nested i:j:k i machines j oper k rep

```
options nodate nonumber ls=80 nocenter FORMDLIM=' ';
```

```
data nested; input machine $ oper $ rsp; cards;
```

```
Machine1 Oper1 79
Machine1 Oper1 62
Machine1 Oper2 94
Machine1 Oper2 74
Machine1 Oper3 46
Machine1 Oper3 57
Machine2 Oper4 92
Machine2 Oper4 99
Machine2 Oper5 85
Machine2 Oper5 79
Machine2 Oper6 76
Machine2 Oper6 68
Machine3 Oper7 88
Machine3 Oper7 75
Machine3 Oper8 53
Machine3 Oper8 56
Machine3 Oper9 46
Machine3 Oper9 57
Machine4 Oper10 36
Machine4 Oper10 53
Machine4 Oper11 40
Machine4 Oper11 56
Machine4 Oper12 62
Machine4 Oper12 47
```

```
; run;
```

```
proc glm data = nested;
```

```
class machine oper;
```

```
model rsp = machine oper(machine);
```

```
random machine oper(machine) / test;
```

```
run;
```

```
proc mixed data = nested cl;
```

```
class machine oper;
```

```
model rsp = ;
```

```
random machine oper(machine);
```

```
run; quit;
```

Assignment #6 SKETCH OF SOULTIONS

4

Class Level Information

Class	Levels	Values
machine	4	Machine1 Machine2 Machine3 Machine4
oper	3	Oper1 Oper2 Oper3

Number of Observations Read	24
Number of Observations Used	24

Source	Type III Expected Mean Square
machine	Var(Error) + 2 Var(oper(machine)) + 6 Var(machine)
oper(machine)	Var(Error) + 2 Var(oper(machine))

Dependent Variable: rsp

Source	DF	Type III SS	Mean Square	F Value	Pr > F
machine	3	3617.666667	1205.888889	3.42	0.0728 <- insufficient evidence of machine effect
Error	8	2817.666667	352.208333		
Error: MS(oper(machine))					

Source	DF	Type III SS	Mean Square	F Value	Pr > F
oper(machine)	8	2817.666667	352.208333	4.17	0.0134 <- evidence of oper effect
Error: MS(Error)	12	1014.000000	84.500000		

Point Estimation & Interval Estimation of Variance Components

Cov Parm	Estimate	Alpha	Lower	Upper
machine	142.28	0.05	33.5679	18486
oper(machine)	133.85	0.05	50.0000	942.04
Residual	84.5000	0.05	43.4509	230.26