1)

First, assign the numbers 1-15 to the 15 wholeplots. Create a random permutation of the numbers 1-15 and for the first 3 numbers (plots) in the random permutation, assign wholeplot treatment 1 *A1*, for the second 3 numbers (plots), assign wholeplot treatment 2 *A2*, …, etc. Within each wholeplot, label the experimental units 1,2,3,4. Create a random permutation of these numbers to assign the B subplot treatments.

The model is as follows:

$y_{ijk} = \mu + \tau_i + \delta_{ik} + \gamma_j + \tau\gamma_{ij} + \epsilon_{ijk}$; $i = 1,2,3,4,5$; $j = 1,2,3,4$; $k = 1,2,3$

τ_i is the fixed effect of the i^{th} level of A

γ_j is the fixed effect of the j^{th} level of B

$\tau\gamma_{ij}$ is the fixed interaction effect of the i^{th} level of A on the j^{th} level of B

δ_{ik} is the random effect for the k^{th} wholeplot receiving the i^{th} level of A. The δ_{ik} are independent normal with mean 0 and variance σ_δ^2

ϵ_{ijk} is the random error effect. They are independent normals with mean 0 and variance σ_ϵ^2 .

The AOV table is shown below:

Source	SS	df	EMS
A	SSA	4	$\sigma_\epsilon^2 + 4\sigma_\delta^2 + 12\theta_\tau$
Wholeplot Error	SS(A)	10	$\sigma_\epsilon^2 + 4\sigma_\delta^2$
B	SSB	3	$\sigma_\epsilon^2 + 15\theta_\gamma$
AB	SSAB	12	$\sigma_\epsilon^2 + 3\theta_{\tau\gamma}$
Subplot Error	SSE	30	σ_ϵ^2
Total	SSTot	59	

2)

For each of the three blocks, assign each experimental unit (wholeplot) a number 1, 2, 3, 4, 5. Obtain a random permutation of the numbers 1, 2, 3, 4, 5 and assign treatment 1 *A1* to the first, 2 *A2* to the second, …, etc. Once the wholeplots have been randomized in the blocks, assign each subplot a number 1, 2, 3, 4. Obtain a random permutation of those numbers 1, 2, 3, 4 and assign treatment 1 *B1* to the first, 2 *B2* to the second, and so on.

The model is as follows:

$y_{ijk} = \mu + \tau_i + \beta_j + \tau\beta_{ij} + \gamma_k + \tau\gamma_{ik} + \epsilon_{ijk}$; $i = 1,2,3,4,5$; $j = 1,2,3$; $k = 1,2,3,4$

τ_i is the fixed effect of the i^th level of A

β_j is the block effect of the j^th block

$\tau\beta_{ij}$ is the interaction between the i^th level of A and the j^th block effect

γ_k is the fixed effect of the k^th level of B

$\tau\gamma_{ik}$ is the fixed interaction effect of the i^th level of A on the k^th level of B

The AOV table is shown below:

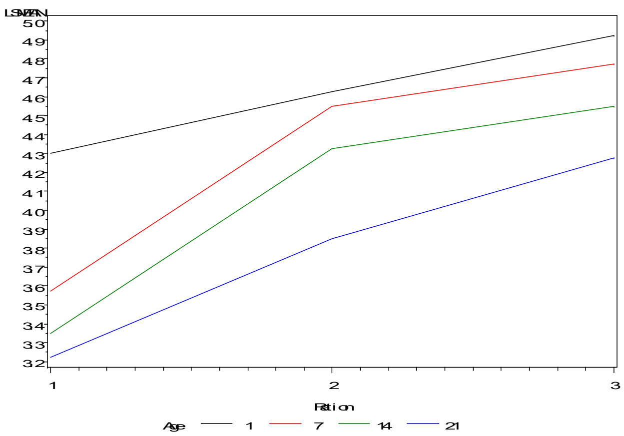
Source	SS	df	EMS
Block	SSBlock	2	sigmasq_error + 4sigmasq_ABlk + 20sigmasq_Blk
A	SSA	4	$\sigma_\epsilon^2 + 4\sigma_{\tau\beta}^2 + 12\theta_\tau$
A*Block	SSA*Block	8	$\sigma_\epsilon^2 + 4\sigma_{\tau\beta}^2$
B	SSB	3	$\sigma_\epsilon^2 + 15\theta_\gamma$
AB	SSAB	12	$\sigma_\epsilon^2 + 3\theta_{\tau\gamma}$
Error	SSE	30	σ_ϵ^2
Total	SSTot	59	

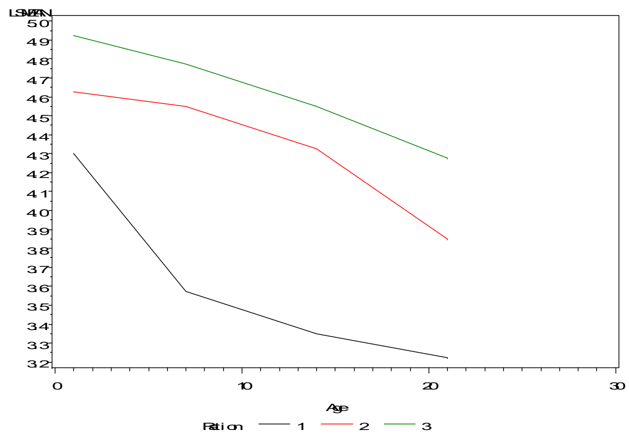
3)

```
proc glm;
class  Ration     Steer Age;
model ShearForce = Ration Steer(Ration) Age Age*Ration;
random Steer(Ration) / test;
lsmeans age*ration /out=abmeans;
run;

symbol i=j;

proc gplot data=abmeans;
plot lsmean*age=ration;
plot lsmean*ration=age;
run;
```





It appears the decrease in mean shear force with increased aging of the steaks is similar under all rations. Ration 1 acts somewhat differently (quicker drop off between 1 and 7 days), but overall, the plot suggests that the interaction term is not significant.

3) continues …

The model is as follows:

$y_{ijk} = \mu + \tau_i + \delta_{ik} + \gamma_j + \tau\gamma_{ij} + \varepsilon_{ijk}$; $i = 1,2,3$; $j = 1,2,3,4$; $k = 1,2,\ldots,\boxed{4}$ where

τ_i is the fixed effect of ration

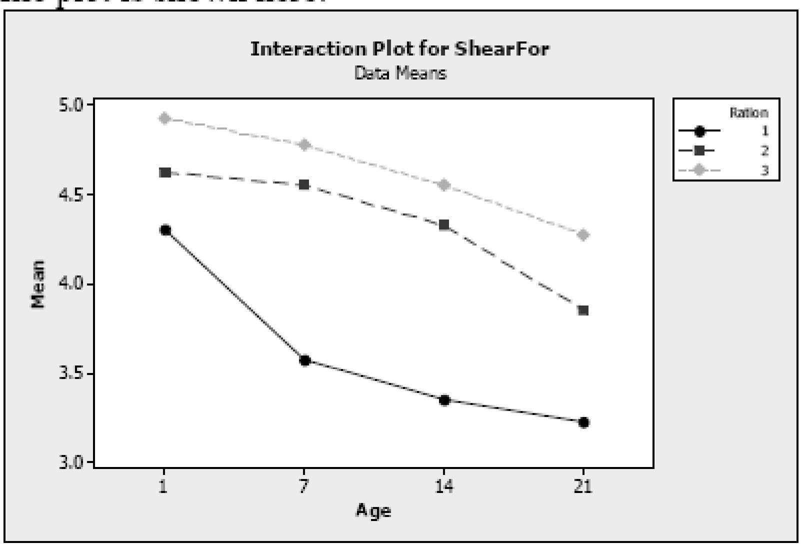
δ_{ik} is the random effect of the k^{th} steer receiving ration i

γ_j is the fixed effect of the j^{th} age

$\tau\gamma_{ij}$ is the fixed interaction effect of the i^{th} ration with the j^{th} age

ε_{ijk} is the random effect due to all other sources of variation

The profile plot is shown here:



It appears the decrease in mean shear force with increased aging of the steaks is similar under all rations. Ration 1 acts somewhat differently (quicker drop off between 1 and 7 days), but overall, the plot suggests that the interaction term is not significant.

3) continues …

```
The GLM Procedure
Dependent Variable: y
                                       Sum of
Source                      DF        Squares     Mean Square    F Value    Pr > F
Model                       20     52.25750000      2.61287500      47.97    <.0001
Error                       27      1.47062500      0.05446759
Corrected Total             47     53.72812500

R-Square     Coeff Var      Root MSE        y Mean
0.972628      5.565018      0.233383      4.193750

Source                      DF    Type III SS     Mean Square    F Value    Pr > F
RATION                       2     8.79875000      4.39937500      80.77    <.0001
RATION*REP                   9    38.26187500      4.25131944      78.05    <.0001
AGE                          3     4.47229167      1.49076389      27.37    <.0001
AGE*RATION                   6     0.72458333      0.12076389       2.22    0.0722
Split-Plot Design                                                               22

The GLM Procedure
Source                 Type III Expected Mean Square
RATION                 Var(Error) + 4 Var(RATION*REP) + Q(RATION,AGE*RATION)
RATION*REP             Var(Error) + 4 Var(RATION*REP)
AGE                    Var(Error) + Q(AGE,AGE*RATION)
AGE*RATION             Var(Error) + Q(AGE*RATION)

The GLM Procedure
Tests of Hypotheses for Mixed Model Analysis of Variance
Dependent Variable: y
     Source                 DF    Type III SS     Mean Square    F Value    Pr > F
*    RATION                  2       8.798750        4.399375       1.03    0.3940
     Error                   9      38.261875        4.251319
Error: MS(RATION*REP)
* This test assumes one or more other fixed effects are zero.

     Source                 DF    Type III SS     Mean Square    F Value    Pr > F
     RATION*REP              9      38.261875        4.251319      78.05    <.0001
*    AGE                     3       4.472292        1.490764      27.37    <.0001
     AGE*RATION              6       0.724583        0.120764       2.22    0.0722
     Error: MS(Error)       27       1.470625        0.054468
* This test assumes one or more other fixed effects are zero.
```

b. The p-value to test for the significance of the interaction is 0.0722 which implies the interaction between ration and age is not significant. This will allow inference on the main effects.

c. The p-value to test the significance of the effect of rations is $0.3940 > 0.05$. Therefore, it can be concluded that there is not a significant effect of ration on mean shear force.

d. The p-value to test the significance of the effect of age is < 0.0001. Therefore, it can be concluded that there is a significant effect of age on mean shear force.

3) continues …

a. To perform this experiment as a completely randomized design of the same number of samples, we would need 48 cows. First, assign a random number to the 48 cows and order them. For the first four on the list, assign ration 1, age 1, the next four, ration 1, age 7, and so on until all cows assigned.

b. The gain in running the study as a completely randomized design is a large increase in the degrees of freedom for testing the ration effect.

c. The split-plot design was chosen to significantly reduce the number of steers from 48 to 12. In the split plot design, there are fewer degrees of freedom for testing the ration effect, but the gain in using fewer steers may be worth the cost.