Addition and Subtraction with Polynomials

MATH 101 *College Algebra*

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Objectives

In this lesson we will learn to:

- identify polynomial expressions,
- classify certain polynomials as monomials, binomials, or trinomials,
- add and subtract polynomials,
- evaluate polynomials for given values of the variables.

Monomials

Definition

A **monomial in** x is an expression of the form

$$kx^n$$

where n is a positive whole number and k is any real number. Exponent n is called the **degree** of the monomial and k is called the **coefficient**.

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Comments:

- A monomial may have more than one variable. In this case the degree is the sum of the exponents of the variables.
- A nonzero constant monomial has degree 0 (since $k = k \cdot x^0$).
- The constant 0 has no degree.

Polynomials

Definition

A **polynomial** is a monomial or the algebraic sum or difference of monomials.

The **degree of a polynomial** is the largest of the degrees of its terms after like terms have been combined.

The coefficient of the term with the largest degree is called the **leading coefficient**.

Polynomials with one, two, or three terms are called monomials, **binomials**, and **trinomials** respectively.

Polynomials of degree 0 or 1 are called **linear**, of degree 2 are called **quadratic**, and of degree 3 are called **cubic**.

Notation

We will write polynomials left to right in descending order of degree, for example

$$3x + 5x^2 - 10 + \frac{1}{3}x^3 = \frac{1}{3}x^3 + 5x^2 + 3x - 10.$$

Adding Polynomials

Add polynomials by combining like terms.

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Example

$$(3x^2 - 2x + 5) + (2x^2 - x + 3)$$

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Example

$$
\begin{aligned}
&(3x^2 - 2x + 5) + (2x^2 - x + 3) \\
&= 3x^2 + 2x^2 - 2x - x + 5 + 3 \\
&= 5x^2 - 3x + 8
\end{aligned}
$$

Subtracting Polynomials

To find the difference of two polynomials add the opposite of each term being subtracted.

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$$(5x^3 - 9x^2 - 10x + 12) - (3x^3 + 6x^2 - 7)$$

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Example

$$
\begin{aligned}
&(5x^3 - 9x^2 - 10x + 12) - (3x^3 + 6x^2 - 7) \\
&= 5x^3 - 3x^3 - 9x^2 - 6x^2 - 10x + 12 + 7 \\
&= 2x^3 - 15x^2 - 10x + 19
\end{aligned}
$$

Evaluation of Polynomials

We will sometimes use function notation when dealing with polynomials.

$$P(x) = x^3 - 8x^2 - 5x + 10$$
$$P(x, y) = 2x^2y - xy + 3x - 4y + 1$$

These functions are read as "P of x" and as "P of x and y".

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To evaluate the polynomial at a number, substitute the number for the variable of the polynomial.