

Introduction to Functions

MATH 101 *College Algebra*

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Objectives

- ▶ Find the domain and range of a function.
- ▶ Determine whether a relation is a function.
- ▶ Use the vertical line test to determine whether a graph is the graph of a function.
- ▶ Express functions using proper functional notation.

Relations

Definition

- ▶ A **relation** is a set of ordered pairs of real numbers.
- ▶ The **domain**, D , of a relation is the set of all first coordinates in the relation.
- ▶ The **range**, R , of a relation is the set of all second coordinates in the relation.

Remark: When graphing relations we will place **domain** elements on the **horizontal axis** and **range** elements on the **vertical axis**.

Functions

Definition

A **function** is a relation in which each domain element is paired with exactly one corresponding range element.

Remarks:

- ▶ A relation is a function if each first coordinate appears only once.
- ▶ A relation is a function if no two ordered pairs have the same first coordinate.

Vertical Line Test

Remark: if we graph two points with the same first coordinate, the points will appear directly above and below each other, in other words, on the same vertical line.

Definition

If **any** vertical line intersects the graph of a relation at more than one point, then the relation is **not** the graph of a function.

Linear Functions

Definition

A **linear function** is a function represented by an equation of the form

$$y = mx + b.$$

The domain of a linear function is the set of all real numbers, $D = (-\infty, \infty)$.

Remarks:

- ▶ A vertical line is not a graph of a function.
- ▶ If the line is not horizontal, the range is all real numbers, $R = (-\infty, \infty)$.
- ▶ If the line is horizontal, the range is $R = \{b\}$.

Non-linear Functions

If we are given a function stated as a formula and no domain is explicitly stated, **the domain will be assumed to be the set of all real x -values for which the formula is defined.**

When determining the domain of a function, remember that:

- ▶ no denominator can equal 0, and
- ▶ negative numbers are not allowed under square roots.

Function Notation

For ease of use we often give letter names to functions.

The linear function $y = mx + b$ may instead be written as

$$f(x) = mx + b$$

where the symbols $f(x)$ are read as “ f of x ”.

If we replace the symbol x by a real number, we are **evaluating** the function.