# Multiplication and Division of Complex Numbers

MATH 101 *College Algebra*

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# Objectives

In this lesson we will learn to:

- multiply with complex numbers,
- divide with complex numbers, and
- simplify powers of $i$.

# Multiplication of Complex Numbers

For complex numbers $a + bi$ and $c + di$ we define their product to be

$$(a + bi)(c + di) = (ac - bd) + (ad + bc)i.$$

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This product formula can be verified using the FOIL method.

$$\begin{aligned}(a + bi)(c + di) &= ac + adi + bci + bdi^2 \\ &= ac + (ad + bc)i + bd(-1) \\ &= (ac - bd) + (ad + bc)i\end{aligned}$$

# A Common Error

Remember that $\sqrt{a}\sqrt{b} = \sqrt{a\,b}$ only when $a > 0$ and $b > 0$. Following this rule helps us avoid errors like the following.

$$\sqrt{-3}\sqrt{-6} \neq \sqrt{(-3)(-6)} = \sqrt{18} = 3\sqrt{2}$$

Instead we use complex numbers to multiply.

$$\sqrt{-3}\sqrt{-6} = i\sqrt{3}\,i\sqrt{6} = i^2\sqrt{18} = -3\sqrt{2}$$

# Division with Complex Numbers

Given the complex numbers $a + bi$ and $c + di$ we find the product

$$\frac{a + bi}{c + di}$$

by multiplying the numerator and denominator by the **complex conjugate** of the denominator.

$$\begin{aligned}\frac{a + bi}{c + di} &= \frac{(a + bi)(c - di)}{(c + di)(c - di)} \\ &= \frac{(ac + bd) + (bc - ad)i}{c^2 + d^2} \\ &= \frac{ac + bd}{c^2 + d^2} + i\frac{bc - ad}{c^2 + d^2}\end{aligned}$$

# Powers of $i$ (1 of 2)

$$
\begin{aligned}
i^1 &= i \\
i^2 &= -1 \\
i^3 &= i^2\, i = -i \\
i^4 &= i^3\, i = 1 \\
i^5 &= i^4\, i = i \\
i^6 &= i^5\, i = -1 \\
i^7 &= i^6\, i = -i \\
i^8 &= i^7\, i = 1 \\
&\vdots
\end{aligned}
$$

**Note:** the powers of $i$ can take on only one of four different values.

# Powers of $i$ (2 of 2)

We can summarize the powers of $i$ in the following way:

$$\begin{aligned} i^{4n} &= 1 \\ i^{4n+1} &= i^{4n}\, i = i \\ i^{4n+2} &= i^{4n}\, i^2 = -1 \\ i^{4n+3} &= i^{4n}\, i^3 = -i \end{aligned}$$

To simplify a power of $i$, divide the exponent $k$ by 4 and keep the remainder $r$, then

$$i^k = i^r.$$