Multiplication with Polynomials

MATH 101 *College Algebra*

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Objectives

In this lesson we will learn to:

- multiply polynomials using the distributive property,
- multiply two binomials using the FOIL method,
- multiply binomials, finding products that are the difference of squares, and
- square binomials, finding products that are perfect square trinomials.

Multiplication by a Monomial

The **distributive property** states that

$$a(b+c) = ab + ac \quad \text{and} \quad (b+c)a = ba + ca.$$

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$$\begin{aligned} 4xy^2(x^2 + 3x - 2) &= (4xy^2)(x^2) + (4xy^2)(3x) - (4xy^2)(2) \\ &= 4x^3y^2 + 12x^2y^2 - 8xy^2 \end{aligned}$$

Multiplication by a Polynomial

We can multiply two polynomials by using the distributive property more than once.

Example

$$\begin{aligned}(3x+1)(2x-5) &= (3x)(2x-5)+(1)(2x-5)\\ &= (3x)(2x)-(3x)(5)+2x-5\\ &= 6x^2-15x+2x-5\\ &= 6x^2-13x-5\end{aligned}$$

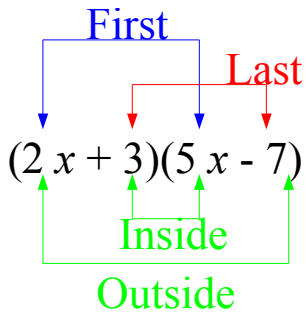
FOIL Method

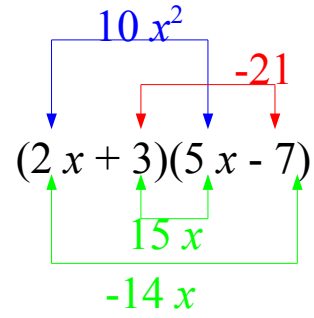
The **FOIL** method is a memory aid for helping you multiply two binomials.

Example

$$
\begin{aligned}
(2x+3)(5x-7) &= (2x)(5x-7)+(3)(5x-7) \\
&= \underbrace{(2x)(5x)}_{\textbf{F}\text{irst}} + \underbrace{(2x)(-7)}_{\textbf{O}\text{utside}} + \underbrace{(3)(5x)}_{\textbf{I}\text{nside}} + \underbrace{(3)(-7)}_{\textbf{L}\text{ast}}
\end{aligned}
$$

Picture the FOIL Method





$$(2x+3)(5x-7) = 10x^2 - 14x + 15x - 21 = 10x^2 + x - 21$$

Difference of Two Squares

Definition

A product of two binomials of the form $(x + a)(x - a)$ is called the **difference of two squares** since

$$(x + a)(x - a) = x^2 - a^2.$$

Check: by the FOIL method

$$(x + a)(x - a) = x^2 + ax - ax - a^2 = x^2 - a^2.$$

Perfect Square Trinomials

Definition

A product of the form

- $(x + a)^2$ is a called a **square of a binomial sum**, and
- $(x - a)^2$ is a called a **square of a binomial difference**.

The products are

$$(x + a)^2 = x^2 + 2ax + a^2$$
$$(x - a)^2 = x^2 - 2ax + a^2.$$

Since each product is a trinomial, these are called **perfect square trinomials**.

Common Errors

Since we have seen that

$$(x+a)^2 = (x+a)(x+a) = x^2 + 2ax + a^2$$
$$(x-a)^2 = (x-a)(x-a) = x^2 - 2ax + a^2$$

NEVER, EVER MAKE THE MISTAKES BELOW.

$$(x+a)^2 \neq x^2 + a^2$$
$$(x-a)^2 \neq x^2 - a^2$$