Operations with Radicals

MATH 101 *College Algebra*

J Robert Buchanan

Department of Mathematics

Fall 2022

Objectives

In this lesson we will learn to

- perform arithmetic operations with radical expressions, and
- rationalize the denominators of radicals.

Addition and Subtraction of Radicals

We can add or subtract radical expressions as long as they have the same index and radicand.

Example

- $\sqrt{13} - 3\sqrt{13} = (1-3)\sqrt{13} = -2\sqrt{13}$
- $4\sqrt{12} + \sqrt{75} = 4(2)\sqrt{3} + 5\sqrt{3} = 8\sqrt{3} + 5\sqrt{3} = 13\sqrt{3}.$

Multiplication with Radical Expressions

We may often use the FOIL method to multiply expressions involving radicals.

Example

$$(\sqrt{3}+\sqrt{8})(\sqrt{2}+1)=\sqrt{6}+\sqrt{3}+\sqrt{16}+\sqrt{8}=\sqrt{6}+\sqrt{3}+4+2\sqrt{2}$$

Rationalizing the Denominators of Radical Expressions

- Often we will encounter rational expressions (fractions) with radicals in their denominators.
- It is generally easier to work with a rational expression if any radicals present appear only in the numerator.
- The process of re-writing a rational expression with a radical in the denominator in an equivalent form without radicals in the denominator is called **rationalizing the denominator**.

Rationalizing the Denominator

Steps:

1. If the denominator contains a square root, multiply both the numerator and the denominator by an expression that will give a denominator with no square roots.
2. If the denominator contains a cube root, multiply both the numerator and the denominator by an expression that will give a denominator with no cube roots.

Rationalizing the Denominator

Steps:

1. If the denominator contains a square root, multiply both the numerator and the denominator by an expression that will give a denominator with no square roots.
2. If the denominator contains a cube root, multiply both the numerator and the denominator by an expression that will give a denominator with no cube roots.

Example

$$\frac{3+\sqrt{5}}{2\sqrt{3}} =$$

Rationalizing the Denominator

Steps:

1. If the denominator contains a square root, multiply both the numerator and the denominator by an expression that will give a denominator with no square roots.
2. If the denominator contains a cube root, multiply both the numerator and the denominator by an expression that will give a denominator with no cube roots.

Example

$$\frac{3+\sqrt{5}}{2\sqrt{3}} = \frac{(3+\sqrt{5})}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} =$$

Rationalizing the Denominator

Steps:

1. If the denominator contains a square root, multiply both the numerator and the denominator by an expression that will give a denominator with no square roots.
2. If the denominator contains a cube root, multiply both the numerator and the denominator by an expression that will give a denominator with no cube roots.

Example

$$\frac{3+\sqrt{5}}{2\sqrt{3}} = \frac{(3+\sqrt{5})}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{(3+\sqrt{5})\sqrt{3}}{2\sqrt{3}\sqrt{3}} =$$

Rationalizing the Denominator

Steps:

1. If the denominator contains a square root, multiply both the numerator and the denominator by an expression that will give a denominator with no square roots.
2. If the denominator contains a cube root, multiply both the numerator and the denominator by an expression that will give a denominator with no cube roots.

Example

$$\frac{3+\sqrt{5}}{2\sqrt{3}} = \frac{(3+\sqrt{5})}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{(3+\sqrt{5})\sqrt{3}}{2\sqrt{3}\sqrt{3}} = \frac{3\sqrt{3}+\sqrt{15}}{6}$$

Rationalizing a Sum or Difference of Radicals

If the denominator of a rational expression involves the sum or difference of square roots we will employ the **difference of squares special factoring form**.

Definition

The two expressions $a + b$ and $a - b$ are called **conjugates** of each other.

Rationalizing a Sum or Difference of Radicals

If the denominator of a rational expression involves the sum or difference of square roots we will employ the **difference of squares special factoring form**.

Definition

The two expressions $a + b$ and $a - b$ are called **conjugates** of each other.

Remark: the product of conjugates is always the difference of two squares.

Rationalizing the Denominator

If the denominator of a fraction contains a sum or difference involving a square root, rationalize the denominator by multiplying both the numerator and denominator by the **conjugate of the denominator**.

Steps:

1. If the denominator is of the form $a - b$, multiply both numerator and denominator by $a + b$.
2. If the denominator is of the form $a + b$, multiply both numerator and denominator by $a - b$.

The new denominator will be the difference of two squares and therefore will not contain any radicals.

Example

$$\frac{\sqrt{5}-3\sqrt{2}}{\sqrt{6}+\sqrt{10}} = \frac{(\sqrt{5}-3\sqrt{2})(\sqrt{6}-\sqrt{10})}{(\sqrt{6}+\sqrt{10})(\sqrt{6}-\sqrt{10})}$$

$$= \frac{\sqrt{30}-3\sqrt{12}-\sqrt{50}+3\sqrt{20}}{6-10}$$

$$= \frac{\sqrt{30}-6\sqrt{3}-5\sqrt{2}+6\sqrt{5}}{-4}$$

$$= \frac{6\sqrt{3}+5\sqrt{2}-6\sqrt{5}-\sqrt{30}}{4}$$