Special Factoring Techniques

MATH 101 *College Algebra*

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Objectives

In this lesson we will learn to:

- factor the difference of two squares,
- factor perfect square trinomials, and
- factor the sum and difference of two cubes.

Products of Binomials

You should recall these product formulas.

$$(x+a)(x-a) = x^2 - a^2 \quad \text{(difference of squares)}$$

$$(x+a)^2 = x^2 + 2ax + a^2 \quad \text{(square of binomial sum)}$$

$$(x-a)^2 = x^2 - 2ax + a^2 \quad \text{(square of binomial difference)}$$

Difference of Squares

Given a polynomial $x^2 - a^2$ we recognize this as the difference of two squares and we factor it directly as

$$x^2 - a^2 = (x + a)(x - a).$$

Remarks:

- We may need to factor out a common monomial first.
- The sum of squares is not factorable.

Perfect Square Trinomials

There are two types:

$$x^2 + 2ax + a^2 = (x + a)^2$$
$$x^2 - 2ax + a^2 = (x - a)^2.$$

The first and last terms of the trinomial must be perfect squares.

If the first term is x^2 and the last term is a^2 then the middle term must be either

$$2ax \qquad \text{or} \qquad -2ax.$$

Sums and Differences of Cubes

If we use the distributive property we see that

$$\begin{aligned}(x+a)(x^2-ax+a^2) &= x(x^2-ax+a^2)+a(x^2-ax+a^2)\\ &= x^3-ax^2+a^2x+ax^2-a^2x+a^3\\ &= x^3+a^3 \quad \text{(sum of cubes)}\end{aligned}$$

and

$$\begin{aligned}(x-a)(x^2+ax+a^2) &= x(x^2+ax+a^2)-a(x^2+ax+a^2)\\ &= x^3+ax^2+a^2x-ax^2-a^2x-a^3\\ &= x^3-a^3 \quad \text{(difference of cubes).}\end{aligned}$$