Probability Rules

MATH 130, *Elements of Statistics I*

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Introduction

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- If we know that in a population of 2000 telephone numbers, 1380 are for cell phones and 620 are land lines, then picking a telephone number at random yields a probability of

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  of picking a cell phone.
- If the proportion of cell phones in a population is unknown then by sampling we may infer the proportion of cell phones.

Experimental *vs.* Empirical Probability

Experiment: toss (or simulate the toss) a fair coin 1000 times and record the outcomes.

Outcome	**Freq.**	**Rel. Freq.**
Heads	506	0.506
Tails	494	0.494

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In the long run the proportion of heads should be $1/2$.

Rolling Dice

Experiment: roll (or simulate the roll) a pair of fair dice 1000 times and record the outcomes.

Outcome	Freq.	Rel. Freq.
2	19	0.019
3	46	0.046
4	93	0.093
5	95	0.095
6	147	0.147
7	187	0.187
8	129	0.129
9	121	0.121
10	76	0.076
11	67	0.067
12	20	0.020

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Outcome	Freq.	Rel. Freq.	Long Run
2	19	0.019	0.0278
3	46	0.046	0.0556
4	93	0.093	0.0833
5	95	0.095	0.1111
6	147	0.147	0.1389
7	187	0.187	0.1667
8	129	0.129	0.1389
9	121	0.121	0.1111
10	76	0.076	0.0833
11	67	0.067	0.0556
12	20	0.020	0.0278

Law of Large Numbers

Definition

Law of Large Numbers: as the number of repetitions of an experiment increases, the proportion with which a certain outcome is observed gets closer to the empirical probability of the outcome.

Terminology (1 of 2)

Definition

An **experiment** is a process with uncertain results that yields an observation.

Definition

An **outcome** is a particular result of an experiment.

Terminology (2 of 2)

Definition

The **sample space**, S of an experiment is the collection of all possible outcomes.

Definition

An **event** is any collection of outcomes from an experiment. An event may consist of more than one outcome. Events will be denoted E, while events consisting of a single outcome are called **simple events** and will be denoted e_i.

Example

An experiment consists of rolling a single fair die.

1. What are the outcomes?

2. What is the sample space?

3. Are the outcomes simple events?
4. Describe the event E = “roll an odd number”.

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$$E = \{1, 3, 5\}$$

Rules of Probability

Rules:

1. The probability of an event E will be denoted $P(E)$, must be greater than or equal to 0 and less than or equal to 1.

$$0 \leq P(E) \leq 1$$

2. The sum of the probabilities of all the outcomes must equal 1. If $S = \{e_1, e_2, \ldots, e_n\}$ then

$$P(e_1) + P(e_2) + \cdots + P(e_n) = \sum P(e_i) = 1.$$

Impossible and Unusual

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Remark: generally if the probablity of an event is less than 0.05 or 5%, the event is considered unusual.

Empirical Probability

Definition

The probability of an event E is approximated by the relative frequency of occurrence of the event in an experiment.

$$P(E) \approx \frac{\text{frequency of } E}{\text{number of experimental trials}}$$

Example

Ten fair coins are tossed 1000 times and the number of heads is observed. What is the approximate probability of obtaining exactly 3 heads when tossing 10 coins.

Outcome	Frequency
0	0
1	9
2	48
3	109
4	210
5	242
6	213
7	128
8	32
9	9
10	0

Classical Method

If an experiment has n equally likely outcomes and if the number of ways that an event E can occur is m, then

$$P(E) = \frac{\text{number of ways } E \text{ can occur}}{\text{number of possible outcomes}} = \frac{m}{n}.$$

Remark: knowing how many outcomes an experiment has and whether the outcomes are equally likely is something we will spend time studying.

Example (1 of 2)

Find the probability of each outcome of the rolling of a pair of fair dice using the classical method.

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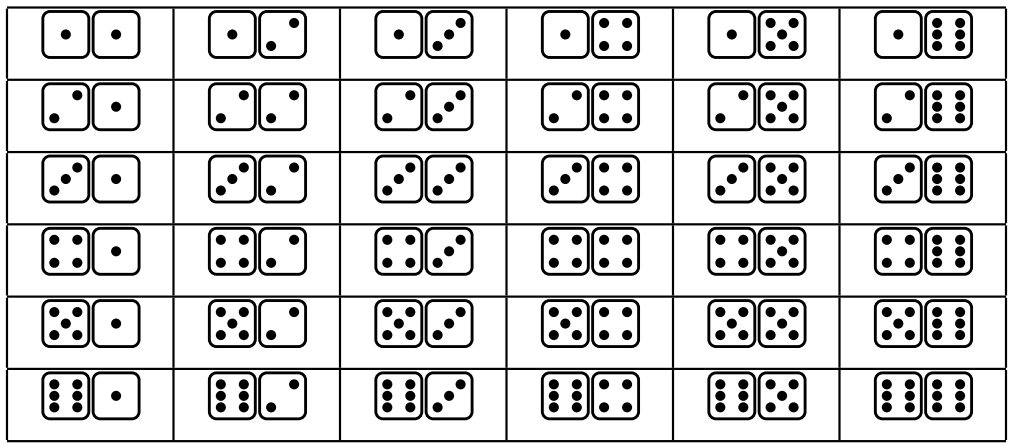

Example (2 of 2)

Dice Totals

2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9
5	6	7	8	9	10
6	7	8	9	10	11
7	8	9	10	11	12

Example (2 of 2)

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4	5	6	7	8	9
5	6	7	8	9	10
6	7	8	9	10	11
7	8	9	10	11	12

E	$P(E)$
2	0.0278
3	0.0556
4	0.0833
5	0.1111
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Example

On days with a high temperature of 85°F and humidity of 90% there is a 0.65 probability of rain (65% chance of rain).