Probability Rules

MATH 130, *Elements of Statistics I*

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Objectives

- Determine the appropriate probability rule to use.
- Determine the appropriate counting technique to use.

Rules of Probability

Addition Rule for Disjoint Events: $P(A \text{ or } B) = P(A) + P(B)$

General Addition Rule: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Multiplication Rule for Independent Events:

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

General Multiplication Rule: $P(A \text{ and } B) = P(A) \cdot P(B|A)$

Complements Rule: $P(A^c) = 1 - P(A)$.

Rules for Counting

- If a task consists of a sequence of choices in which there are p selections for the first choice, q selections for the second choice, r selections for the third choice, and so on, then the task of making these selections can be done in

$$p \cdot q \cdot r \cdots$$

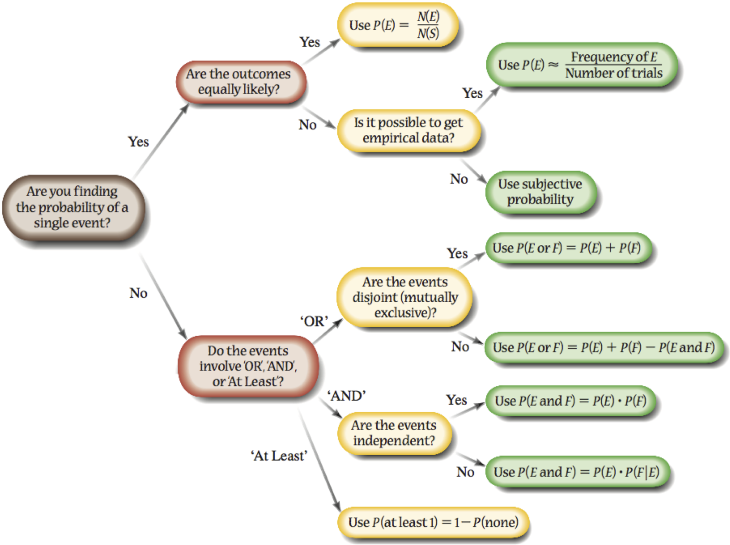
  different ways.
- The number of combinations of n different objects taken r at a time.

$${}_nC_r = \frac{n!}{r!(n-r)!}$$

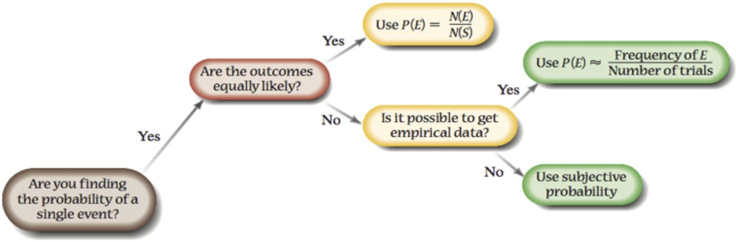
- The number of permutations of r objects selected from n objects.

$${}_nP_r = \frac{n!}{(n-r)!}$$

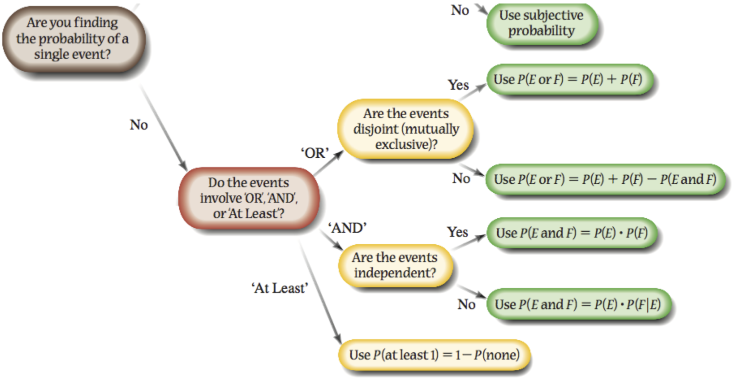
Global Flow Chart



Single Event Flow Chart



Compound Event Flow Chart



Deal or No Deal?

In the game show *Deal or No Deal*?, a contestant is presented with 26 suitcases that contain amounts ranging from $0.01 to $1,000,000. The contestant must pick an initial case that is set aside as the game progresses. The amounts are randomly distributed among the suitcases prior to the game. Consider the following breakdown:

Prize	**Number of Suitcases**
$0.01–$100	8
$200–$1000	6
$5000–$50,000	5
$100,000–$1,000,000	7

What is the probability of picking a case worth at least $100,000?

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$$P(\text{prize} \geq \$100,000) = \frac{7}{26} = 0.2692$$

Tattoos

According to a Harris poll in January 2008, 14% of adult Americans have one or more tattoos, 50% have pierced ears, and 65% of those with one or more tattoos also have pierced ears. What is the probability that a randomly selected adult American has one or more tattoos and pierced ears?

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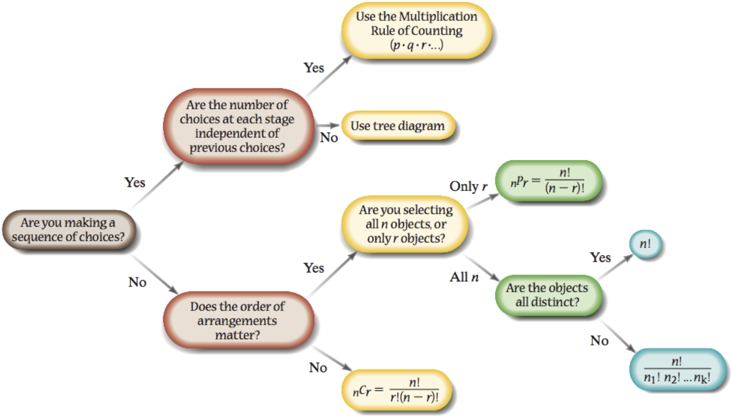
The poll results state

$$\begin{aligned} P(\text{tattoo}) &= 0.14 \\ P(\text{pierced ears}) &= 0.50 \\ P(\text{pierced ears} \mid \text{tattoo}) &= 0.65. \end{aligned}$$

Using the Multiplication Rule,

$$\begin{aligned} P(\text{tattoo and pierced ears}) &= P(\text{tattoo}) \cdot P(\text{pierced ears} \mid \text{tattoo}) \\ &= (0.14)(0.65) = 0.091. \end{aligned}$$

Counting Techniques



Example

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The number of subcommittees that can be formed is

$$({}_5C_3)({}_4C_2) = \left(\frac{5!}{3!(5-3)!}\right)\left(\frac{4!}{2!(4-2)!}\right) = (10)(6) = 60.$$

Combinations are used since the ordering of members of the subcommittee is irrelevant.

Example

On February 17, 2008, the Daytona International Speedway hosted the 50th running of the Daytona 500. Touted by many to be the most anticipated event in racing history, the race carried a record purse of almost $18.7 million. With 43 drivers in the race, in how many different ways could the top four finishers (1st, 2nd, 3rd, and 4th place) occur?

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The number of top four finishing orders is

$$_{43}P_4 = \frac{43!}{(43-4)!} = \frac{43!}{39!} = (43)(42)(41)(40) = 2,961,840.$$

Permutations were used since the finishing order is relevant.