Neville's Method

MATH 375 *Numerical Analysis*

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Motivation

- We have learned how to approximate a function using Lagrange polynomials and how to estimate the error in such an approximation.
- Sometimes we want to approximate a function at a particular point x rather than along an interval $[a, b]$.
- Today we will learn how to interpolate between data values found in a table **without** knowing the function that generated the values.
- We will see that we can perform the interpolation **without** explicitly writing out the interpolating polynomial.

Lagrange Polynomials

Definition

If f is a function defined at $x_0, x_1, \ldots, x_n$ distinct real numbers and $m_1, m_2, \ldots, m_k$ are k distinct integers with $0 \leq m_i \leq n$, then the Lagrange polynomial that agrees with f at the k points $x_{m_1}, x_{m_2}, \ldots, x_{m_k}$ is denoted $P_{m_1, m_2, \cdots, m_k}$.

Example (1 of 3)

Let $x_0 = 1$, $x_1 = 2$, $x_2 = 4$, $x_3 = 5$, and $x_4 = 7$ and $f(x) = J_0(x)$ (**Bessel function** of the first kind of order zero).

Determine $P_{1,3,4}(x)$.

Example (1 of 3)

Let $x_0 = 1$, $x_1 = 2$, $x_2 = 4$, $x_3 = 5$, and $x_4 = 7$ and $f(x) = J_0(x)$ (**Bessel function** of the first kind of order zero).

Determine $P_{1,3,4}(x)$.

$$
\begin{aligned}
P_{1,3,4}(x) &= \frac{(x-5)(x-7)}{(2-5)(2-7)} J_0(2) + \frac{(x-2)(x-7)}{(5-2)(5-7)} J_0(5) \\
&\quad + \frac{(x-2)(x-5)}{(7-2)(7-5)} J_0(7) \\
&= \left[\frac{1}{15}x^2 - \frac{4}{5}x + \frac{7}{3}\right] J_0(2) + \left[-\frac{1}{6}x^2 + \frac{3}{2}x + \frac{7}{3}\right] J_0(5) \\
&\quad + \left[\frac{1}{10}x^2 - \frac{7}{10}x + 1\right] J_0(7)
\end{aligned}
$$

Example (2 of 3)

Let $x_0 = 1$, $x_1 = 2$, $x_2 = 4$, $x_3 = 5$, and $x_4 = 7$ and $f(x) = J_0(x)$.

Use $P_{1,3,4}(3)$ to approximate $J_0(3)$.

Example (2 of 3)

Let $x_0 = 1$, $x_1 = 2$, $x_2 = 4$, $x_3 = 5$, and $x_4 = 7$ and $f(x) = J_0(x)$.

Use $P_{1,3,4}(3)$ to approximate $J_0(3)$.

$$\begin{aligned}
P_{1,3,4}(x) &= \frac{(x-5)(x-7)}{(2-5)(2-7)} J_0(2) + \frac{(x-2)(x-7)}{(5-2)(5-7)} J_0(5) + \frac{(x-2)(x-5)}{(7-2)(7-5)} J_0(7) \\
P_{1,3,4}(3) &= \frac{(3-5)(3-7)}{(2-5)(2-7)} J_0(2) + \frac{(3-2)(3-7)}{(5-2)(5-7)} J_0(5) + \frac{(3-2)(3-5)}{(7-2)(7-5)} J_0(7) \\
&= \frac{8}{15} J_0(2) + \frac{2}{3} J_0(5) - \frac{1}{5} J_0(7) \\
&\approx \frac{8}{15}(0.223891) + \frac{2}{3}(-0.177597) - \frac{1}{5}(0.300079) \\
&= -0.0590053
\end{aligned}$$

Example (2 of 3)

Let $x_0 = 1$, $x_1 = 2$, $x_2 = 4$, $x_3 = 5$, and $x_4 = 7$ and $f(x) = J_0(x)$.

Use $P_{1,3,4}(3)$ to approximate $J_0(3)$.

$$\begin{aligned}
P_{1,3,4}(x) &= \frac{(x-5)(x-7)}{(2-5)(2-7)} J_0(2) + \frac{(x-2)(x-7)}{(5-2)(5-7)} J_0(5) + \frac{(x-2)(x-5)}{(7-2)(7-5)} J_0(7) \\
P_{1,3,4}(3) &= \frac{(3-5)(3-7)}{(2-5)(2-7)} J_0(2) + \frac{(3-2)(3-7)}{(5-2)(5-7)} J_0(5) + \frac{(3-2)(3-5)}{(7-2)(7-5)} J_0(7) \\
&= \frac{8}{15} J_0(2) + \frac{2}{3} J_0(5) - \frac{1}{5} J_0(7) \\
&\approx \frac{8}{15}(0.223891) + \frac{2}{3}(-0.177597) - \frac{1}{5}(0.300079) \\
&= -0.0590053
\end{aligned}$$

Actual value: $J_0(3) \approx -0.260052$

Example (3 of 3)

Let $x_0 = 1$, $x_1 = 2$, $x_2 = 4$, $x_3 = 5$, and $x_4 = 7$ and $f(x) = J_0(x)$.

Use a simplified form of $P_{1,3,4}(3)$ to approximate $J_0(3)$.

Example (3 of 3)

Let $x_0 = 1$, $x_1 = 2$, $x_2 = 4$, $x_3 = 5$, and $x_4 = 7$ and $f(x) = J_0(x)$.

Use a simplified form of $P_{1,3,4}(3)$ to approximate $J_0(3)$.

$$
\begin{aligned}
P_{1,3,4}(x) &= \frac{(x-5)(x-7)}{(2-5)(2-7)} J_0(2) + \frac{(x-2)(x-7)}{(5-2)(5-7)} J_0(5) + \frac{(x-2)(x-5)}{(7-2)(7-5)} J_0(7) \\
&= \left(\frac{x^2}{15} - \frac{4x}{5} + \frac{7}{3} \right) J_0(2) + \left(-\frac{x^2}{6} + \frac{3x}{2} - \frac{7}{3} \right) J_0(5) + \left(\frac{x^2}{10} - \frac{7x}{10} + 1 \right) J_0(7) \\
&= \left(\frac{J_0(2)}{15} - \frac{J_0(5)}{6} + \frac{J_0(7)}{10} \right) x^2 + \left(-\frac{4J_0(2)}{5} + \frac{3J_0(5)}{2} - \frac{7J_0(7)}{10} \right) x \\
&\quad + \left(\frac{7J_0(2)}{3} - \frac{7J_0(5)}{3} + J_0(7) \right)
\end{aligned}
$$

$P_{1,3,4}(3) \approx -0.0590053$

Actual value: $J_0(3) \approx -0.260052$

Comments

$$P_{1,3,4}(x) = \frac{(x-5)(x-7)}{(2-5)(2-7)} J_0(2) + \frac{(x-2)(x-7)}{(5-2)(5-7)} J_0(5) + \frac{(x-2)(x-5)}{(7-2)(7-5)} J_0(7)$$

$$P_{1,3,4}(x) = \left(\frac{J_0(2)}{15} - \frac{J_0(5)}{6} + \frac{J_0(7)}{10} \right) x^2 + \left(-\frac{4J_0(2)}{5} + \frac{3J_0(5)}{2} - \frac{7J_0(7)}{10} \right) x$$
$$+ \left(\frac{7J_0(2)}{3} - \frac{7J_0(5)}{3} + J_0(7) \right)$$

- The first form is easier to modify if we add more interpolation nodes, but more difficult to evaluate.
- The second form is easier to evaluate, but more difficult to modify if new interpolation nodes are added.

Interpolation from Data

Suppose we had only the following table of data for $f(x)$:

x	$f(x)$
10.1	0.17537
22.2	0.37784
32.0	0.52992
41.6	0.66393
50.5	0.63608

Approximate $f(27.5)$ using polynomial interpolation.

Interpolation from Data

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22.2	0.37784
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50.5	0.63608

Approximate $f(27.5)$ using polynomial interpolation.
Linear approximation:

$$P_1(27.5) = \frac{(27.5 - 32.0)}{(22.2 - 32.0)} f(22.2) + \frac{(27.5 - 22.2)}{(32.0 - 22.2)} f(32.0) \approx 0.46009$$

Higher Order Approximations

Quadratic approximation: (two choices)

$$\begin{aligned} P_2(27.5) = & \frac{(27.5-22.2)(27.5-32.0)}{(10.1-22.2)(10.1-32.0)} f(10.1) \\ & + \frac{(27.5-10.1)(27.5-32.0)}{(22.2-10.1)(22.2-32.0)} f(22.2) \\ & + \frac{(27.5-10.1)(27.5-22.2)}{(32.0-10.1)(32.0-22.2)} f(32.0) \approx 0.46141 \\ \hat{P}_2(27.5) = & \frac{(27.5-32.0)(27.5-41.6)}{(22.2-32.0)(22.2-41.6)} f(22.2) \\ & + \frac{(27.5-22.2)(27.5-41.6)}{(32.0-22.2)(32.0-41.6)} f(32.0) \\ & + \frac{(27.5-22.2)(27.5-32.0)}{(41.6-22.2)(41.6-32.0)} f(41.6) \approx 0.46200 \end{aligned}$$

There are also two potential cubic interpolating polynomials and a single quartic polynomial.

Remarks

- Without knowing $f(x)$ we have no idea of the size of the errors in these approximations.
- The highest degree polynomial does not necessarily deliver the smallest error.
- Knowing the Lagrange Interpolating Polynomial of degree k does not help us find the one of degree $k + 1$.
- Neville's method (forthcoming) enables the interpolating functions to be built recursively.

Theoretical Result

Theorem

Let f be defined at $x_0, x_1, \ldots, x_k$ and x_i and x_j be two distinct real numbers in this set. Then the kth degree Lagrange polynomial that interpolates f at $x_0, x_1, \ldots, x_k$ is

$$P(x) = \frac{(x - x_j)}{(x_i - x_j)} P_{0,1,\ldots,j-1,j+1,\ldots,k}(x) - \frac{(x - x_i)}{(x_i - x_j)} P_{0,1,\ldots,i-1,i+1,\ldots,k}(x).$$

Theoretical Result

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$$P(x) = \frac{(x - x_j)}{(x_i - x_j)} P_{0,1,\ldots,j-1,j+1,\ldots,k}(x) - \frac{(x - x_i)}{(x_i - x_j)} P_{0,1,\ldots,i-1,i+1,\ldots,k}(x).$$

This theorem allows us to build a Lagrange polynomial of degree k from two Lagrange polynomials each of degree $k - 1$.

Proof

Let $Q(x) = P_{0,1,\ldots,j-1,j+1,\ldots,k}(x)$ and $\hat{Q}(x) = P_{0,1,\ldots,i-1,i+1,\ldots,k}(x)$, then

$$P(x) = \frac{(x - x_j)Q(x) - (x - x_i)\hat{Q}(x)}{(x_i - x_j)}.$$

Proof

Let $Q(x) = P_{0,1,\ldots,j-1,j+1,\ldots,k}(x)$ and $\hat{Q}(x) = P_{0,1,\ldots,i-1,i+1,\ldots,k}(x)$, then

$$P(x) = \frac{(x - x_j)Q(x) - (x - x_i)\hat{Q}(x)}{(x_i - x_j)}.$$

If $x = x_i$, then $P(x_i) = Q(x_i) = f(x_i)$. If $x = x_j$, then $P(x_j) = \hat{Q}(x_j) = f(x_j)$. If $l \in \{0, 1, \ldots, k\}$ with $i \neq l \neq j$ then

$$\begin{aligned} P(x_l) &= \frac{(x_l - x_j)Q(x_l) - (x_l - x_i)\hat{Q}(x_l)}{(x_i - x_j)} \\ &= \frac{(x_l - x_j)f(x_l) - (x_l - x_i)f(x_l)}{(x_i - x_j)} = f(x_l). \end{aligned}$$

Proof

Let $Q(x) = P_{0,1,\ldots,j-1,j+1,\ldots,k}(x)$ and $\hat{Q}(x) = P_{0,1,\ldots,i-1,i+1,\ldots,k}(x)$, then

$$P(x) = \frac{(x - x_j)Q(x) - (x - x_i)\hat{Q}(x)}{(x_i - x_j)}.$$

If $x = x_i$, then $P(x_i) = Q(x_i) = f(x_i)$. If $x = x_j$, then $P(x_j) = \hat{Q}(x_j) = f(x_j)$. If $l \in \{0, 1, \ldots, k\}$ with $i \neq l \neq j$ then

$$\begin{aligned} P(x_l) &= \frac{(x_l - x_j)Q(x_l) - (x_l - x_i)\hat{Q}(x_l)}{(x_i - x_j)} \\ &= \frac{(x_l - x_j)f(x_l) - (x_l - x_i)f(x_l)}{(x_i - x_j)} = f(x_l). \end{aligned}$$

Thus $P(x)$ is a polynomial of degree at most k which interpolates $f(x)$ at x_i for $i = 0, 1, \ldots, k$ and thus

$$P(x) = P_{0,1,\ldots,k}(x).$$

Recursive Generation of Polynomials

$$P_{0,1}(x) = \frac{(x - x_0)P_1(x) - (x - x_1)P_0(x)}{x_1 - x_0}$$

$$P_{1,2}(x) = \frac{(x - x_1)P_2(x) - (x - x_2)P_1(x)}{x_2 - x_1}$$

$$P_{0,1,2}(x) = \frac{(x - x_0)P_{1,2}(x) - (x - x_2)P_{0,1}(x)}{x_2 - x_0}$$

$$P_{1,2,3}(x) = \frac{(x - x_1)P_{2,3}(x) - (x - x_3)P_{1,2}(x)}{x_3 - x_1}$$

$$P_{0,1,2,3}(x) = \frac{(x - x_0)P_{1,2,3}(x) - (x - x_3)P_{0,1,2}(x)}{x_3 - x_0}$$

and so on ...

Neville's Method

Define polynomial $Q_{i,j}(x) = P_{i,i+1,\ldots,i+j}(x)$ where $0 \leq j \leq i$.

- j represents the degree of the polynomial.
- i determines the starting index of $j+1$ consecutive values of x used.
- $Q_{i,0}(x) = f(x_i)$ for $i = 0, 1, \ldots, n$.
- $Q_{i,j}(x)$ can be found recursively.

$$Q_{i,j}(x) = \frac{(x - x_i)Q_{i+j,j-1}(x) - (x - x_{i+j})Q_{i,j-1}(x)}{x_{i+j} - x_i}$$

Tabular Calculation

Given $x_0, x_1, \ldots, x_n$ and $f(x_0), f(x_1), \ldots f(x_n)$ we can arrange the Lagrange interpolating polynomials in the following table.

$$\begin{array}{llllll}
x_0 & f(x_0) = Q_{0,0} & P_{0,1} = Q_{0,1} & P_{0,1,2} = Q_{0,2} & \cdots & P_{0,1,\ldots,n} = Q_{0,n} \\
x_1 & f(x_1) = Q_{1,0} & P_{1,2} = Q_{1,1} & P_{1,2,3} = Q_{1,2} & & \\
x_2 & f(x_2) = Q_{2,0} & P_{2,3} = Q_{2,1} & P_{2,3,4} = Q_{2,2} & & \\
\vdots & \vdots & \vdots & \vdots & & \\
x_{n-2} & f(x_{n-2}) = Q_{n-2,0} & P_{n-2,n-1} = Q_{n-2,1} & P_{n-2,n-1,n} = Q_{n-2,2} & & \\
x_{n-1} & f(x_{n-1}) = Q_{n-1,0} & P_{n-1,n} = Q_{n-1,1} & & & \\
x_n & f(x_n) = Q_{n,0} & & & &
\end{array}$$

Neville's Iterated Interpolation Algorithm

Goal: evaluate the interpolating polynomial P on $n+1$ distinct numbers $x_0, \ldots, x_n$ at x to approximate $f(x)$.

INPUT values x, abscissas $x_0, \ldots, x_n$, ordinates $f(x_0), \ldots, f(x_n)$

STEP 1 For $i = 0, 1, \ldots, n$ set $Q_{i,0} = f(x_i)$.

STEP 2 For $j = 1, 2, \ldots, n$

For $i = 0, 1, \ldots, n-j$

Set $$Q_{i,j} = \frac{(x - x_i)Q_{i+j,j-1} - (x - x_{i+j})Q_{i,j-1}}{x_{i+j} - x_i}.$$

STEP 3 OUTPUT table Q. STOP.

$Q_{0,n}$ will hold the desired value $P_n(x)$.

Example

Approximate $f(27.5)$ from the following data.

x_i	$f(x_i)$
10.1	0.17537
22.2	0.37784
32.0	0.52992
41.6	0.66393
50.5	0.63608

First task is to order x_i's by increasing distance from $x = 27.5$.

Solution (1 of 8)

The column of ordinate values has been re-headed as $Q_{i,0} = f(x_i)$.

i	x_i	$Q_{i,0}$	$Q_{i,1}$
0	32.0	0.52992	
1	22.2	0.37784	
2	41.6	0.66393	
3	10.1	0.17537	
4	50.5	0.63608	

$$Q_{0,1} = \frac{(x - x_0)Q_{1,0} - (x - x_1)Q_{0,0}}{x_1 - x_0}$$

$$Q_{1,1} = \frac{(x - x_1)Q_{2,0} - (x - x_2)Q_{1,0}}{x_2 - x_1}$$

$$Q_{2,1} = \frac{(x - x_2)Q_{3,0} - (x - x_3)Q_{2,0}}{x_3 - x_2}$$

$$Q_{3,1} = \frac{(x - x_3)Q_{4,0} - (x - x_4)Q_{3,0}}{x_4 - x_3}$$

Solution (2 of 8)

i	x_i	$Q_{i,0}$	$Q_{i,1}$
0	32.0	0.52992	0.46009
1	22.2	0.37784	0.45600
2	41.6	0.66393	0.44524
3	10.1	0.17537	0.37380
4	50.5	0.63608	

$$Q_{0,1} = \frac{(27.5 - 32.0)(0.37784) - (27.5 - 22.2)(0.52992)}{22.2 - 32.0}$$

$$Q_{1,1} = \frac{(27.5 - 22.2)(0.66393) - (27.5 - 41.6)(0.37784)}{41.6 - 22.2}$$

$$Q_{2,1} = \frac{(27.5 - 41.6)(0.17537) - (27.5 - 10.1)(0.66393)}{10.1 - 41.6}$$

$$Q_{3,1} = \frac{(27.5 - 10.1)(0.63608) - (27.5 - 50.5)(0.17537)}{50.5 - 10.1}$$

Solution (3 of 8)

i	x_i	$Q_{i,0}$	$Q_{i,1}$	$Q_{i,2}$
0	32.0	0.52992	0.46009	
1	22.2	0.37784	0.45600	
2	41.6	0.66393	0.44524	
3	10.1	0.17537	0.37380	
4	50.5	0.63608		

$$Q_{0,2} = \frac{(x - x_0)Q_{1,1} - (x - x_2)Q_{0,1}}{x_2 - x_0}$$

$$Q_{1,2} = \frac{(x - x_1)Q_{2,1} - (x - x_3)Q_{1,1}}{x_3 - x_1}$$

$$Q_{2,2} = \frac{(x - x_2)Q_{3,1} - (x - x_4)Q_{2,1}}{x_4 - x_2}$$

Solution (4 of 8)

i	x_i	$Q_{i,0}$	$Q_{i,1}$	$Q_{i,2}$
0	32.0	0.52992	0.46009	0.46200
1	22.2	0.37784	0.45600	0.46071
2	41.6	0.66393	0.44524	0.55843
3	10.1	0.17537	0.37380	
4	50.5	0.63608		

$$Q_{0,2} = \frac{(27.5 - 32.0)(0.45600) - (27.5 - 41.6)(0.46009)}{41.6 - 32.0}$$

$$Q_{1,2} = \frac{(27.5 - 22.2)(0.44524) - (27.5 - 10.1)(0.45600)}{10.1 - 22.2}$$

$$Q_{2,2} = \frac{(27.5 - 41.6)(0.37380) - (27.5 - 50.5)(0.44524)}{50.5 - 41.6}$$

Solution (5 of 8)

i	x_i	$Q_{i,0}$	$Q_{i,1}$	$Q_{i,2}$	$Q_{i,3}$
0	32.0	0.52992	0.46009	0.46200	
1	22.2	0.37784	0.45600	0.46071	
2	41.6	0.66393	0.44524	0.55843	
3	10.1	0.17537	0.37380		
4	50.5	0.63608			

$$Q_{0,3} = \frac{(x - x_0)Q_{1,2} - (x - x_3)Q_{0,2}}{x_3 - x_0}$$

$$Q_{1,3} = \frac{(x - x_1)Q_{2,2} - (x - x_4)Q_{1,2}}{x_4 - x_1}$$

Solution (6 of 8)

i	x_i	$Q_{i,0}$	$Q_{i,1}$	$Q_{i,2}$	$Q_{i,3}$
0	32.0	0.52992	0.46009	0.46200	0.46174
1	22.2	0.37784	0.45600	0.46071	0.47901
2	41.6	0.66393	0.44524	0.55843	
3	10.1	0.17537	0.37380		
4	50.5	0.63608			

$$Q_{0,3} = \frac{(27.5 - 32.0)(0.46071) - (27.5 - 10.1)(0.46200)}{10.1 - 32.0}$$

$$Q_{1,3} = \frac{(27.5 - 22.2)(0.55843) - (27.5 - 50.5)(0.46071)}{50.5 - 22.2}$$

Solution (7 of 8)

i	x_i	$Q_{i,0}$	$Q_{i,1}$	$Q_{i,2}$	$Q_{i,3}$	$Q_{i,4}$
0	32.0	0.52992	0.46009	0.46200	0.46174	
1	22.2	0.37784	0.45600	0.46071	0.47901	
2	41.6	0.66393	0.44524	0.55843		
3	10.1	0.17537	0.37380			
4	50.5	0.63608				

$$Q_{0,4} = \frac{(x - x_0)Q_{1,3} - (x - x_4)Q_{0,3}}{x_4 - x_0}$$

Solution (8 of 8)

i	x_i	$Q_{i,0}$	$Q_{i,1}$	$Q_{i,2}$	$Q_{i,3}$	$Q_{i,4}$
0	32.0	0.52992	0.46009	0.46200	0.46174	0.45754
1	22.2	0.37784	0.45600	0.46071	0.47901	
2	41.6	0.66393	0.44524	0.55843		
3	10.1	0.17537	0.37380			
4	50.5	0.63608				

$$Q_{0,4} = \frac{(27.5 - 32.0)(0.47901) - (27.5 - 50.5)(0.46174)}{50.5 - 32.0}$$

Example

Use Neville's Iterated Interpolation to obtain the approximations for Lagrange interpolating polynomials of degrees one, two, and three for $f(0)$.

x_i	$f(x_i)$
-0.50	1.93750
-0.25	1.33203
0.25	0.80078
0.50	0.68750

Solution (1 of 3)

Sort the abscissas in order of proximity to $x = 0$.

i	x_i	$f(x_i)$	$Q_{i,1}$
0	-0.25	1.33203	
1	0.25	0.80078	
2	-0.50	1.93750	
3	0.50	0.68750	

Solution (1 of 3)

Sort the abscissas in order of proximity to $x = 0$.

i	x_i	$f(x_i)$	$Q_{i,1}$
0	-0.25	1.33203	
1	0.25	0.80078	
2	-0.50	1.93750	
3	0.50	0.68750	

$$Q_{0,1} = \frac{(0-(-0.25))0.80078 - (0-0.25)1.33203}{0.25-(-0.25)}$$

$$Q_{1,1} = \frac{(0-0.25)1.93750 - (0-(-0.50))0.80078}{-0.50-0.25}$$

$$Q_{2,1} = \frac{(0-(-0.50))0.68750 - (0-0.50)1.93750}{0.50-(-0.50)}$$

Solution (1 of 3)

Sort the abscissas in order of proximity to $x = 0$.

i	x_i	$f(x_i)$	$Q_{i,1}$
0	-0.25	1.33203	1.06641
1	0.25	0.80078	1.17969
2	-0.50	1.93750	1.31250
3	0.50	0.68750	

$$Q_{0,1} = \frac{(0-(-0.25))0.80078 - (0-0.25)1.33203}{0.25-(-0.25)}$$

$$Q_{1,1} = \frac{(0-0.25)1.93750 - (0-(-0.50))0.80078}{-0.50-0.25}$$

$$Q_{2,1} = \frac{(0-(-0.50))0.68750 - (0-0.50)1.93750}{0.50-(-0.50)}$$

Solution (2 of 3)

Find the interpolation of order 2.

i	x_i	$f(x_i)$	$Q_{i,1}$	$Q_{i,2}$
0	-0.25	1.33203	1.06641	
1	0.25	0.80078	1.17969	
2	-0.50	1.93750	1.31250	
3	0.50	0.68750		

Solution (2 of 3)

Find the interpolation of order 2.

i	x_i	$f(x_i)$	$Q_{i,1}$	$Q_{i,2}$
0	-0.25	1.33203	1.06641	
1	0.25	0.80078	1.17969	
2	-0.50	1.93750	1.31250	
3	0.50	0.68750		

$$Q_{0,2} = \frac{(0-(-0.25))1.17969 - (0-(-0.50))1.06641}{-0.50-(-0.25)}$$

$$Q_{1,2} = \frac{(0-0.25)1.31250 - (0-0.50)1.17969}{0.50-0.25}$$

Solution (2 of 3)

Find the interpolation of order 2.

i	x_i	$f(x_i)$	$Q_{i,1}$	$Q_{i,2}$
0	-0.25	1.33203	1.06641	0.953123
1	0.25	0.80078	1.17969	1.04687
2	-0.50	1.93750	1.31250	
3	0.50	0.68750		

$$Q_{0,2} = \frac{(0-(-0.25))1.17969 - (0-(-0.50))1.06641}{-0.50-(-0.25)}$$

$$Q_{1,2} = \frac{(0-0.25)1.31250 - (0-0.50)1.17969}{0.50-0.25}$$

Solution (3 of 3)

Find the interpolation of order 3.

i	x_i	$f(x_i)$	$Q_{i,1}$	$Q_{i,2}$	$Q_{i,3}$
0	-0.25	1.33203	1.06641	0.953123	
1	0.25	0.80078	1.17969	1.04687	
2	-0.50	1.93750	1.31250		
3	0.50	0.68750			

Solution (3 of 3)

Find the interpolation of order 3.

i	x_i	$f(x_i)$	$Q_{i,1}$	$Q_{i,2}$	$Q_{i,3}$
0	-0.25	1.33203	1.06641	0.953123	
1	0.25	0.80078	1.17969	1.04687	
2	-0.50	1.93750	1.31250		
3	0.50	0.68750			

$$Q_{0,3} = \frac{(0-(-0.25))1.04687 - (0-0.50)0.953123}{0.50-(-0.25)}$$

Solution (3 of 3)

Find the interpolation of order 3.

i	x_i	$f(x_i)$	$Q_{i,1}$	$Q_{i,2}$	$Q_{i,3}$
0	-0.25	1.33203	1.06641	0.953123	0.984373
1	0.25	0.80078	1.17969	1.04687	
2	-0.50	1.93750	1.31250		
3	0.50	0.68750			

$$Q_{0,3} = \frac{(0-(-0.25))1.04687 - (0-0.50)0.953123}{0.50-(-0.25)}$$

Homework

- Read Section 3.2.
- Exercises: 1a, 3, 5, 7