Topics in Applied Mathematics: Calculus of Variations
Fall 2014
MATH 478.01 (3 credits), MWF 1:00P-1:50P, Wickersham 219

Prerequisites: A grade of C- or better in MATH 311 (*Calculus III*) is a prerequisite for this course.

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Office Hours: 10:00A-10:50A (MWF), 1:00-1:50P (TuTh), or by appointment
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Textbook: *Calculus of Variations*, Robert Weinstock, Dover Publications, Inc., Mineola, NY, 1974, ISBN: 0-486-63069-2.

Objectives: The Calculus of Variations is an important branch of optimization that deals with finding extrema of functionals in certain functional spaces. It has deep connections with various fields in the natural sciences, including differential geometry, ordinary and partial differential equations, materials science, mathematical biology, *etc.* It is one of the oldest and yet one of the most used tools for investigation of the problems involving free energy. During this semester students will be introduced to the calculus of variations and use it to solve basic problems arising in physics, mathematics, and materials science. The goal of this course is to present the basics of the calculus of variations, including the one-dimensional theory and its application to various problems arising in the natural sciences.

After completing this course students will:

- give an account of the foundations of calculus of variations and of its applications in mathematics and physics,
- understand the basics of the calculus of variations,
- describe the brachistochrone problem mathematically and solve it,
- solve isoperimetric problems of standard type,
- solve simple initial and boundary value problems by using several variable calculus,
- formulate maximum principles for various equations and derive consequences,
- be able to analyze and solve various variational problems arising in the physical sciences,
- be able to present mathematical arguments to others.

Course Contents: The Calculus of Variations deals with optimization problems where the variables, instead of being finite dimensional as in ordinary calculus, are functions. This course treats the foundations of calculus of variations and gives examples on some (classical and modern) applications within physics and engineering science.

- Euler-Lagrange equation.
- Brachistochrone problem.
- Minimal surfaces of revolution.
- Isoperimetric problem.

- Fermat's principle (geometric optics).
- Hamilton's principle (particle dynamics).
- Lagrange's and Hamilton's equations of motion.
- Hamilton-Jacobi equation.
- Principle of least action.

Topics covered in this course will include the following from the textbook.

- Background and Preliminaries (review of multivariable calculus), Chap. 2
- Introductory Problems and solutions, Chap. 3
- Isoperimetric Problems, Chap. 4
- Geometrical Optics, Chap. 5
- Dynamics of Particles, Chap. 6

If time permits other topics may be covered as well.

Attendance: Students are expected to attend all class meetings per the University Approved Guidelines. If you know beforehand that you will be absent from class on the day an assignment is due, you must complete and hand in the assignment prior to the absence. If you are unexpectedly absent the day that an assignment is due you must hand in the assignment at the beginning of the class hour on the first day that you return to class. If you know you will be absent on the day of a test, you must notify me before the time the test is scheduled in order to schedule a make-up test. Students who miss a test should provide a valid excuse, otherwise you will not be allowed to make up the test. No final exam exemptions.

Homework: Students are expected to do their homework and participate in class. Students should expect to spend a minimum of three hours outside of class on homework and review for every hour spent in class. Homework exercises help students review and reinforce concepts covered in class. The textbook exercises are arranged in generally increasing level of difficulty. Working only the low-numbered exercises will not prepare a student sufficiently for the test and final examination exercises. All assigned homework exercises must be worked until successful completion. Other homework exercises may be distributed on paper handouts, sent as email attachments, or posted under Desire2Learn.

Tests: There will be three 50-minute in-class tests and a final examination. Dates for the tests have been set as follows.

- Test #1, Wednesday, September 24, 2014
- Test #2, Friday, October 24, 2014
- Test #3, Monday, November 24, 2014
- Final Examination, Wednesday, December 10, 2014, 10:15A-12:15P

I will not "curve" test or exam grades.

Grades: Course grade will be calculated as follows.

Homework	30%
Tests	50%
Exam	20%

Tests and the final examination will be graded individually on a 100-point scale. I keep a record of students' test, homework, and exam scores. Students should also keep a record of graded assignments, tests, and other materials.

The course letter grade will be assigned as follows. I will not "curve" course grades.

90-92	A−	93-100	A		
80-82	B−	83-86	B	87-89	B+
70-72	C−	73-76	C	77-79	C+
60-62	D−	63-66	D	67-69	D+
		0-59	F		

Course Repeat Policy: An undergraduate student may not take an undergraduate course of record more than three times. A course of record is defined as a course in which a student receives a grade of A, B, C, D, (including + and −) F, U, Z or W. The academic department offering a course may drop a student from a course if the student attempts to take a course more than three times.[1]

The last day to withdraw from a course (and receive the W grade) is Friday, October 31, 2014.

Inclement Weather Policy: If we should miss a class day due to a school delay or cancellation, any activities planned for that missed day will take place the next time the class meets. For example, if a test is scheduled for a day that class is canceled on account of snow, the test will be given the next time the class meets.

Final Word: Mathematics is not a spectator sport. What you learn from this course and your final grade depend mainly on the amount of work you put forth. Daily contact with the material through homework assignments and review of notes taken during lectures is extremely important. Organizing and conducting regular study sessions with other students in this class will help you to understand the material better.

No one can guarantee you success in this course. Your responsibilities and the instructor's expectation are outlined above. There will be no second chances, "do-overs", or extra credit assignments.

[1] Memorandum to mathematics faculty from Dr. Charles G. Denlinger, Assistant Chair, Department of Mathematics, August 30, 2004.