p.957 (12.4)

Find equations of the tangent plane and normal line to the surface at the given point.

5. $z = x^2 + y^2 - 1$ at (2, 1, 4)
7. $z = e^{-x^2-y^2}$ at (0, 0, 1)
9. $z = \sin x \cos y$ at $(0, \pi, 0)$
11. $z = x^3 - 2xy$ at (-2, 3, 4)
13. $z = \sqrt{x^2 + y^2}$ at (-3, 4, 5)
15. $z = \dfrac{4x}{y}$ at (1, 2, 2)

Compute the linear approximation of the function at the given point.

17. $f(x, y) = \sqrt{x^2 + y^2}$ at (3, 0)
19. $f(x, y) = \sin x \cos y$ at $(0, \pi)$
21. $f(x, y) = xe^{y^2} - 4x$ at (2, 0)
23. $f(x, y, z) = \sin yz^2 + x^3 z$ at (-2, 0, 1)
25. $f(w, x, y, z) = w^2 xy - e^{wyz}$ at (-2, 3, 1, 0)

Compare the linear approximation from the indicated exercise to the exact function value at the given points.

27. Exercise 17 at (3, -0.1), (3.1, 0), (3.1, -0.1)
29. Exercise 19 at (0, 3), $(0.1, \pi)$, (0.1, 3)

31. Use a linear approximation to estimate the range of sags in the beam of example 4.5 (found in book) if the error tolerances are $L = 36 \pm 0.5$, $w = 2 \pm 0.2$ and $h = 6 \pm 0.5$.

32. Use a linear approximation to estimate the range of sags in the beam of example 4.5 (found in book) if the error tolerances are $L = 32 \pm 0.4$, $w = 2 \pm 0.3$ and $h = 8 \pm 0.4$.

Find the increment Δz and write it in the form given in Theorem 4.2.

37. $f(x, y) = 2xy + y^2$
39. $f(x, y) = x^2 + y^2$
41. Determine whether or not $f(x, y) = x^2 + 3xy$ is differentiable.
42. Determine whether or not $f(x, y) = xy^2$ is differentiable.

Find the total differential of *f*(*x*, *y*).

43. $f(x, y) = ye^x + \sin x$
44. $f(x, y) = \sqrt{x + y}$

Show that the partial derivative of $f_x(0, 0)$ and $f_y(0, 0)$ both exist, but the function $f(x, y)$ is not differentiable at (0, 0).

45. $f(x,y)=\begin{cases}\dfrac{2xy}{x^2+y^2}, & \text{if } (x,y)\neq(0,0)\\ 0, & \text{if } (x,y)=(0,0)\end{cases}$

46. $f(x,y)=\begin{cases}\dfrac{xy^2}{x^2+y^2}, & \text{if } (x,y)\neq(0,0)\\ 0, & \text{if } (x,y)=(0,0)\end{cases}$

56. Show that $\left\langle 1,\, 0,\, \dfrac{\partial f}{\partial y}(a,b)\right\rangle \times \left\langle 1,\, 0,\, \dfrac{\partial f}{\partial x}(a,b)\right\rangle = \left\langle \dfrac{\partial f}{\partial x}(a,b),\, \dfrac{\partial f}{\partial y}(a,b),\, -1\right\rangle$.