p. 966 (12.5)

Use chain rule to find the indicated derivative(s).

5. $g'(t)$ where $g(t) = f(x(t), y(t))$, $f(x,y) = x^2 y - \sin y$, $x(t) = \sqrt{t^2+1}$, $y(t) = e^t$

6. $g'(t)$ where $g(t) = f(x(t), y(t))$, $f(x,y) = \sqrt{x^2+y^2}$, $x(t) = \sin t$, $y(t) = t^2 + 2$

7. $\frac{\partial g}{\partial u}$ and $\frac{\partial g}{\partial v}$ where $g(u,v) = f(x(u,v), y(u,v))$, $f(x,y) = 4x^2 y^3$, $x(u,v) = u^3 - v$, $y(u,v) = 4u^2$

8. $\frac{\partial g}{\partial u}$ and $\frac{\partial g}{\partial v}$ where $g(u,v) = f(x(u,v), y(u,v))$, $f(x,y) = xy^3 - 4x^2$, $x(u,v) = e^{u^2}$, $y(u,v) = \sqrt{v^2+1}\sin u$

13. In Example 5.2 (in book), suppose that $l = 4$ and $k = 6$, the labor force is decreasing at the rate of 60 workers per year and capital is growing at the rate of \$100,000 per year. Determine the rate of change of production.

15. Suppose the production of a firm is modeled by $P(k,l) = 16k^{1/3}l^{2/3}$, with k and l defined as in example 5.2 (in book). Suppose that $l = 3$ and $k = 4$, the labor force is increasing at the rate of 80 workers per year and capital is decreasing at the rate of \$200,000 per year. Determine the rate of change of production.

17. For a business product, income is the product of the quantity sold and the price, which we can write as $I = qp$. If the quantity sold increases at a rate of 5% and the price increases at a rate of 3%, show that income increases at a rate of 8%.

Use implicit differentiation to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

23. $3x^2 z + 2z^3 - 3yz = 0$

24. $xyz - 4y^2 z^2 + \cos xy = 0$

25. $3e^{xyz} - 4xz^2 + x\cos y = 2$

26. $3yz^2 - e^{4x}\cos 4z - 3y^2 = 4$

27. For a differentiable function $f(x,y)$ with $x = r\cos\theta$ and $y = r\sin\theta$, show that $f_\theta = -f_x r\sin\theta + f_y r\cos\theta$.

31. A baseball player who has h hits in b at bats has a batting average of $a = \frac{h}{b}$. For example, 100 hits in 400 bats would be an average of 0.250. It is traditional to carry three decimal places and to describe this average as being "250 points." To use the chain rule to estimate the change in batting average after a player gets a hit, assume that h and b are functions of time and that getting a hit means $h' = b' = 1$. Show that

$a' = \dfrac{b-h}{b^2}$. Early in a season, a typical batter might have 50 hits in 200 bats. Show that getting a hit will increase batting average by about 4 points. Find the approximate increase in batting average later in the season for a player with 100 hits in 400 bats. In general, if b and h are both doubled, how does a' change?

32. For the baseball player of exercise 31, approximate the number of points that the batting average will decrease by making an out.