

(\* Here's another example showing that the unit normal always points to the concave side of the curve. Consider the graph of the cubic  $y = x^3$  thought of as a parametrized curve with  $x=t$  and  $y=t^3$ . \*)

In[8]:=  $r[t_] := \{t, t^3\}$

(\* The unit tangent is: \*)

In[39]:=  $T = \left\{ \frac{1}{\sqrt{1+9t^4}}, \frac{3t^2}{\sqrt{1+9t^4}} \right\};$

(\* The derivative of the unit tangent is: \*)

In[31]:=  $N1 = \partial_t T$

Out[31]=  $\left\{ -\frac{18t^3}{(1+9t^4)^{3/2}}, -\frac{54t^5}{(1+9t^4)^{3/2}} + \frac{6t}{\sqrt{1+9t^4}} \right\}$

(\* Evaluating at  $t=1$  gives: \*)

In[37]:=  $\left\{ -\frac{18t^3}{(1+9t^4)^{3/2}}, -\frac{54t^5}{(1+9t^4)^{3/2}} + \frac{6t}{\sqrt{1+9t^4}} \right\} // . t \rightarrow 1$

In[38]:=  $N\left[\left\{-\frac{9}{5\sqrt{10}}, 3\sqrt{\frac{2}{5}} - \frac{27}{5\sqrt{10}}\right\}\right]$

Out[38]=  $\{-0.56921, 0.189737\}$

(\* So at (1,1) the unit normal, which has the same direction as the unit tangent, has negative x-component and positive y-component. \*)

(\* Evaluating at  $t=-1$  gives: \*)

In[33]:=  $\left\{ -\frac{18t^3}{(1+9t^4)^{3/2}}, -\frac{54t^5}{(1+9t^4)^{3/2}} + \frac{6t}{\sqrt{1+9t^4}} \right\} // . t \rightarrow -1$

In[35]:=  $N\left[\left\{\frac{9}{5\sqrt{10}}, -3\sqrt{\frac{2}{5}} + \frac{27}{5\sqrt{10}}\right\}\right]$

Out[35]=  $\{0.56921, -0.189737\}$

(\* So at (-1,-1) the unit normal has positive x-component and negative y-component.

In either case, the unit normal points to the concave side of the curve. \*)