MATH 261 – CALCULUS III

REVIEW OF FACTS AND FORMULAS FROM CHAPTER 10

Let $\mathbf{a} = \langle a_1, a_2, \ldots, a_n \rangle$; $\mathbf{b} = \langle b_1, b_2, \ldots, b_n \rangle$.

$t\mathbf{a} = \langle ta_1, ta_2, \ldots, ta_n \rangle$, $t \in \mathbb{R}$

$\mathbf{a} + \mathbf{b} = \langle a_1 + b_1, a_2 + b_2, \ldots, a_n + b_n \rangle$

$\mathbf{a} \bullet \mathbf{b} = \|\mathbf{a}\| \ \|\mathbf{b}\| \ \cos\theta = a_1 b_1 + \cdots + a_n b_n$

$\|\mathbf{a}\| = \sqrt{\mathbf{a} \bullet \mathbf{a}} = \sqrt{a_1^2 + \cdots + a_n^2}$

Unit vector in direction of $\mathbf{a}$ is: $\frac{\mathbf{a}}{\|\mathbf{a}\|}$

$comp_{\mathbf{b}}\mathbf{a} = \mathbf{a} \bullet \frac{\mathbf{b}}{\|\mathbf{b}\|}$

Work $W = \mathbf{F} \bullet \overrightarrow{PQ}$

$$\mathbf{a} \times \mathbf{b} = \left\langle \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}, - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}, \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \right\rangle$$

$\|\mathbf{a} \times \mathbf{b}\| = \|\mathbf{a}\| \ \|\mathbf{b}\| \ \sin\theta$

$\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$

$\mathbf{a} \bullet (\mathbf{b} \times \mathbf{c}) = \mathbf{c} \bullet (\mathbf{a} \times \mathbf{b})$

$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \bullet \mathbf{c})\, \mathbf{b} - (\mathbf{a} \bullet \mathbf{b})\, \mathbf{c}$

$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \times \mathbf{c} + \mathbf{b} \times (\mathbf{a} \times \mathbf{c})$

Volume of parallelopiped determined by $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$: $V = |\mathbf{a} \bullet (\mathbf{b} \times \mathbf{c})|$

Parametric equations for lines ℓ_1 through $P_1(x_1, y_1, z_1)$ with direction $\langle a, b, c \rangle$:

$$\ell_1 : \begin{cases} x = x_1 + at \\ y = y_1 + bt \\ z = z_1 + ct \end{cases}, \ t \in \mathbb{R}$$

If P is any point on line ℓ_1 with direction $\mathbf{a}$
and Q is any point on ℓ_2 is a line with direction $\mathbf{b}$,
the distance from ℓ_1 to ℓ_2 is:

$$Dist = \left| comp_{\mathbf{a}\times\mathbf{b}} \overrightarrow{PQ} \right|$$

Angles between non-parallel lines ℓ_1 and ℓ_2 as above are:

$$\theta = \cos^{-1}\left(\frac{\mathbf{a}\cdot\mathbf{b}}{\|\mathbf{a}\|\,\|\mathbf{b}\|}\right) \text{ and } \pi - \theta$$

$\langle a, b, c\rangle$ is normal to the plane $ax + by + cz + d = 0$.

Non-colinear points $P\left(x_1, y_1, z_1\right)$, Q and R determine the plane:

$$\left(\overrightarrow{PQ} \times \overrightarrow{PR}\right) \cdot \langle x - x_1, y - y_1, z - z_1\rangle = 0$$

If P is any point on plane $ax + by + cz + d = 0$, the
distance from point Q to this plane is:

$$Dist = \left| comp_{\langle a,b,c\rangle} \overrightarrow{PQ} \right|$$