MATH 261 – CALCULUS III

REVIEW OF FACTS AND FORMULAS FROM CHAPTER 11

Position function for a projectile: $\mathbf{r}(t) = (\mathbf{v}_x t + x_0)\,\mathbf{i} + (-16t^2 + \mathbf{v}_y t + y_0)\,\mathbf{j}$

Given a smooth position function: $\mathbf{r}(t) = \langle x(t),\ y(t),\ z(t)\rangle,\ a \le t \le b$:

Velocity: $\mathbf{v} = \mathbf{r}'$

Speed: $v = \|\mathbf{v}\|$

Acceleration: $\mathbf{a} = \mathbf{v}'$

Unit tangent: $\mathbf{T} = \frac{\mathbf{v}}{\|\mathbf{v}\|}$

Unit normal: $\mathbf{N} = \frac{\mathbf{T}'}{\|\mathbf{T}'\|}$

The graph of $\mathbf{r}$ bends towards $\mathbf{N}$.

The 90° counterclockwise and clockwise rotations of $\langle a, b\rangle$ are $\langle -b, a\rangle$ and $\langle b, -a\rangle$, respectively.

Arc length function: $s(t) = \int_a^t \|\mathbf{v}(u)\|\, du$

Curvature if $\mathbf{r}$ is a *unit speed plane curve* $(z = 0)$:

$$K = |\theta'(s)|,\ \text{where } \theta(s) = \cos^{-1}[\mathbf{T}(s) \bullet \mathbf{i}]$$

Curvature if $\mathbf{r}$ is a smooth plane curve $(z = 0)$:

$$K = \frac{|x'y'' - x''y'|}{\left[(x')^2 + (y')^2\right]^{3/2}}$$

Curvature if $\mathbf{r}$ is the graph of a twice differentiable $y = f(x)$:

$$K = \frac{|y''|}{\left[1 + (y')^2\right]^{3/2}}$$

Curvature if $\mathbf{r}$ has *unit speed*: $K = \|\mathbf{T}'\|$

Curvature if $\mathbf{r}$ is a smooth space curve: $K = \frac{\mathbf{a_N}}{v^2} = \frac{\|\mathbf{a}\times\mathbf{v}\|}{\|\mathbf{v}\|^3}$

Radius of curvature: $R = \frac{1}{K}$

Center of curvature is on the normal line to the concave side of the curve.

Tangential component of acceleration: $\mathbf{a_T} = \mathbf{a} \bullet \mathbf{T} = \frac{\mathbf{a}\bullet\mathbf{v}}{\|\mathbf{v}\|} = v'(t)$

Normal component of acceleration: $\mathbf{a_N} = \mathbf{a} \bullet \mathbf{N} = Kv^2 = \frac{\|\mathbf{a}\times\mathbf{v}\|}{\|\mathbf{v}\|} = \sqrt{\|\mathbf{a}\|^2 - \mathbf{a}_\mathbf{T}^2}$