MATH 261 – CALCULUS III

FACTS AND FORMULAS FROM CHAPTER 12

The gradient of f is defined everywhere f has first partials and is given by

$$\nabla f(x,y) = \langle f_x(x,y),\ f_y(x,y)\rangle$$

If f is differentiable at (a,b), then

1. the total derivative of f is

$$f'(x,y) = \nabla f(x,y)$$

2. the tangent plane to the graph of f at $(a,b,f(a,b))$ is the graph of

$$T(x,y) = f(a,b) + f'(a,b) \bullet \langle x-a, y-b\rangle$$

3. the approximate change in f from $(a,b,)$ to (x,y) is

$$df(x,y) = f'(a,b) \bullet \langle dx, dy\rangle$$

4. and $\lim\limits_{(x,y)\to(0,0)} \frac{f(x,y)-T(x,y)}{\|\langle x-a,y-b\rangle\|} = 0.$

Chain rule: $\frac{d}{dt}[f(x(t),y(t))] = f'(x(t),y(t)) \bullet \langle x'(t), y'(t)\rangle$

If y is an implicit differentiable function of x given by $F(x,y)=0$, then

$$\frac{dy}{dx} = -\frac{F_x(x,y)}{F_y(x,y)}$$

The directional derivative of f in the direction of $\mathbf{a}$ is

$$\mathcal{D}_{\mathbf{a}} f(x,y) = \triangledown f(x,y) \bullet \frac{\mathbf{a}}{\|\mathbf{a}\|}$$

If f is differentiable, $\nabla f(a,b) \neq 0$ and $f(a,b)=k$, then $\nabla f(a,b)$ is orthogonal to the level curve $f(x,y)=k$ at (a,b).

If (a, b) is a critical point of f, then either

1. $f_x(a,b) = f_y(a,b) = 0$ or
2. $f_x(a,b)$ or $f_y(a,b)$ does not exist.

If f, f_x, f_y, f_{xy} and f_{yx} are all continuous, then

1. $f_{xy} = f_{yx}$ and
2. the discriminant of f is

$$D(x,y) = \begin{vmatrix} \nabla f_x(x,y) \\ \nabla f_y(x,y) \end{vmatrix} = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = f_{xx} f_{yy} - f_{xy}^2$$

If $\nabla f(a,b) = 0$ and $D(a,b) < 0$, then $(a, b, f(a,b))$ is a saddle point on the graph of f.

If $\nabla f(a,b) = 0$ and $D(a,b) > 0$, then

1. $f(a,b)$ is a local max if $f_{xx}(a,b) < 0$
2. $f(a,b)$ is a local min if $f_{xx}(a,b) > 0$.

If f and g have continuous first partials, $\nabla g \neq 0$ and $f(a,b)$ is an extremum along level curve $g(x,y) = 0$, then $\nabla f(a,b) = \lambda \nabla g(a,b)$.