MATH 261 – CALCULUS III

FACTS AND FORMULAS FROM CHAPTER 13

Let R be a closed and bounded region in the xy-plane. The area A of the region R is:

$$A = \iint_R 1 \; dA.$$

If $\rho(x, y)$ is the density of the lamina R at (x, y), the mass m of R is:

$$m = \iint_R \rho(x, y) \; dA.$$

The first moments M_y and M_x of the lamina R about the x-axis and y-axis are:

$$M_y = \iint_R x\rho(x, y) \; dA \quad \text{and} \quad M_x = \iint_R y\rho(x, y) \; dA.$$

The center of mass $(\bar{x}, \bar{y})$ of the lamina R is:

$$(\bar{x}, \bar{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m} \right).$$

If f is a non-negative continuous function on R, the volume V of the solid under the graph of $z = f(x, y)$ and above the region R is:

$$V = \iint_R f(x, y) \; dA.$$

Let R be a type R_x region, i.e., $R = \{(x, y) : a \leq x \leq b \text{ and } g_1(x) \leq y \leq g_2(x)\}$, where g_1 and g_2 are continuous functions on $[a, b]$. If f is a continuous function on R, then

$$\iint_R f(x, y) \; dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y) \; dy \; dx.$$

Let R be a type R_y region, i.e., $R = \{(x, y) : c \leq x \leq d \text{ and } h_1(y) \leq x \leq h_2(y)\}$, where h_1 and h_2 are continuous functions on $[c, d]$. If f is a continuous function on R, then

$$\iint_R f(x, y) \; dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x, y) \; dx \; dy.$$

Let f be a non-negative function with continuous first partials defined throughout a closed and bounded domain R in the xy-plane. The surface area A of the graph of $z = f(x, y)$ over R is given by

$$A = \iint_R \sqrt{1 + (f_x)^2 + (f_y)^2} \; dA$$

Let $R = \{(r,\theta) : g_1(\theta) \le r \le g_2(\theta) \text{ and } \alpha \le \theta \le \beta\}$ be a region in the polar coordinate plane, where g_1 and g_2 are continuous functions on $[\alpha, \beta]$. If f is a continuous function on R, then

$$\iint_R f(r,\theta)\ dA = \int_\alpha^\beta \int_{g_1(\theta)}^{g_2(\theta)} f(r,\theta)\ r\ dr\ d\theta.$$

Let Q be a closed and bounded solid in space. The volume V of Q is given by

$$V = \iiint_Q 1\ dV.$$

If $\delta(x,y,z)$ is the density of Q at (x,y,z), the mass m of Q is given by

$$m = \iiint_Q \delta(x,y,z)\ dV.$$

Let $Q = \{(x,y,z) : a \le x \le b,\ h_1(x) \le y \le h_2(x),\ \text{and } k_1(x,y) \le z \le k_2(x,y)\}$, where h_1 and h_2 are continous on $[a,b]$ and k_1 and k_2 have continuous first partials. If f is a continuous function on Q, then

$$\iiint_Q f(x,y,z)\ dV = \int_a^b \int_{h_1(x)}^{h_2(x)} \int_{k_1(x,y)}^{k_2(x,y)} f(x,y,z)\ dz\ dy\ dx.$$

To transform between rectangular and cylindrical coordinates:

$$\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \\ z &= z \end{aligned}$$

To transform between rectangular and spherical coordinates:

$$\begin{aligned} x &= \rho\sin\phi\cos\theta \\ y &= \rho\sin\phi\sin\theta \\ z &= \rho\cos\phi \end{aligned}$$

Let $Q = \{(\rho,\phi,\theta) : a \le \rho \le b,\ c \le \phi \le d,\ \text{and } e \le \theta \le f\}$ be a solid region of space in spherical coordinates. If f is continuous on Q, then

$$\iiint_Q f(\rho,\phi,\theta)\ dV = \int_e^f \int_c^d \int_a^b f(\rho,\phi,\theta)\ \rho^2 \sin\phi\ d\rho\ d\phi\ d\theta.$$