MATH 261 – CALCULUS III

FACTS AND FORMULAS FROM CHAPTER 14

Let $\mathbf{r}(t) = \langle x(t), y(t)\rangle$ with $a \le t \le b$ be a piece-wise smooth parametrization of curve C, let f be a differentiable real-valued function on a closed and bounded plane region R and let $\mathbf{F}(x,y) = \langle M(x,y), N(x,y)\rangle$ be a vector field such that M, N, $\frac{\partial N}{\partial x}$, and $\frac{\partial M}{\partial y}$ are continuous on an open region of the plane containing R.

$\int_C f(x,y)\ ds = \int_a^b f(\mathbf{r}(t))\,\|\mathbf{r}'(t)\|\ dt = \int_a^b f(x(t), y(t))\sqrt{[x'(t)]^2 + [y'(t)]^2}\ dt$

$\int_C \mathbf{F}\bullet\ d\mathbf{r} = \int_C \mathbf{F} \bullet \mathbf{T}\ ds = \int_a^b \mathbf{F}(\mathbf{r}(t)) \bullet \mathbf{r}'(t)\ dt$

$\int_C \mathbf{F} \bullet d\mathbf{r}$ is independent of path

iff $\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = 0$

iff there is a real-valued function p such that $\bigtriangledown p = \mathbf{F}$ in which case

$$\int_{(a_1 b_1)}^{(a_2,b_2)} \mathbf{F} \bullet d\mathbf{r} = p(a_2,b_2) - p(a_1,b_1)$$

$\oint_C \mathbf{F} \bullet d\mathbf{r} = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA,$
where C is the boundary of R

$\iint_S g(x,y,z)\ dS = \iint_R g(x,y,f(x,y))\sqrt{(f_x)^2 + (f_y)^2 + 1}\ dA,$
where S is the graph of $z = f(x,y)$

Let $\mathbf{F}(x,y,z) = \langle M(x,y,z), N(x,y,z), P(x,y,z)\rangle$ be a vector field such that M, N, P and their first partials are continuous on an open region of space containing a surface S

$\bigtriangledown = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle$

$div(\mathbf{F}) = \bigtriangledown \bullet \mathbf{F}$

$\mathbf{curl}(\mathbf{F}) = \bigtriangledown \times \mathbf{F}$

$\oint_C \mathbf{F}\bullet\, d\mathbf{r} = \iint_S \mathbf{curl}(\mathbf{F}) \bullet \mathbf{n}\; dS$,
where C is the boundary of S and $\mathbf{n}$ is the upward unit normal on S

$\iint_S \mathbf{F} \bullet \mathbf{n}\; dS = \iiint_Q div(\mathbf{F})\;\, dV$,
where S is the closed piece-wise smooth boundary of Q
and $\mathbf{n}$ is the outward unit normal on S