SELECTED FACTS AND FORMULAS

$\mathbf{a} \bullet \mathbf{b} = \|\mathbf{a}\| \ \|\mathbf{b}\| \ \cos\theta$

$comp_{\mathbf{b}}\mathbf{a} = \mathbf{a} \bullet \frac{\mathbf{b}}{\|\mathbf{b}\|}$

$\|\mathbf{a} \times \mathbf{b}\| = \|\mathbf{a}\| \ \|\mathbf{b}\| \ \sin\theta$

Angles between non-parallel lines with respective direction vectors $\mathbf{a}$ and $\mathbf{b}$:

$$\theta = \cos^{-1}\left(\frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\| \, \|\mathbf{b}\|}\right) \text{ and } \pi - \theta.$$

Let $\mathbf{r}(t) = \langle x(t), \ y(t), \ z(t)\rangle, \ a \le t \le b$ be a smooth position function.

Curvature if $\mathbf{r}$ is a plane curve ($z = 0$): $K = \frac{|x'y'' - x''y'|}{\left[(x')^2 + (y')^2\right]^{3/2}}$

Curvature if $\mathbf{r}$ is the graph of a twice differentiable $y = f(x)$: $K = \frac{|y''|}{\left[1 + (y')^2\right]^{3/2}}$

Curvature if $\mathbf{r}$ has *unit speed*: $K = \|\mathbf{T}'\|$.

Curvature if $\mathbf{r}$ is a space curve: $K = \frac{\|\mathbf{a} \times \mathbf{v}\|}{\|\mathbf{v}\|^3}$.

Tangential component of acceleration: $\mathbf{a_T} = \mathbf{a} \bullet \mathbf{T} = \mathbf{a} \bullet \frac{\mathbf{v}}{\|\mathbf{v}\|}$.

Normal component of acceleration: $\mathbf{a_N} = \mathbf{a} \bullet \mathbf{N} = \mathbf{a} \bullet \frac{\mathbf{T}'}{\|\mathbf{T}'\|} = K\|\mathbf{v}\|^2 = \frac{\|\mathbf{a} \times \mathbf{v}\|}{\|\mathbf{v}\|}$
$= \sqrt{\|\mathbf{a}\|^2 - \mathbf{a}_\mathbf{T}^2}$.

If P is any point on line ℓ_1 with direction $\mathbf{a}$ and Q is any point on ℓ_2 is a line with direction $\mathbf{b}$, the distance from ℓ_1 to ℓ_2 is:

$$Dist = \left|comp_{\mathbf{a} \times \mathbf{b}} \overrightarrow{PQ}\right|.$$

If P is any point on plane $ax + by + cz + d = 0$, the distance from point Q to this plane is:

$$Dist = \left|comp_{\langle a,b,c\rangle} \overrightarrow{PQ}\right|.$$

The first moments M_y and M_x of the lamina R about the x-axis and y-axis are:

$$M_y = \iint_R x\rho(x,y) \ dA \ \text{ and } \ M_x = \iint_R y\rho(x,y) \ dA.$$

The center of mass $(\bar{x}, \bar{y})$ of the lamina R is: $(\bar{x}, \bar{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m}\right)$.

Let $\mathbf{F}(x,y,z) = \langle M(x,y,z), N(x,y,z), P(x,y,z) \rangle$ be a vector field such that M, N, P and their first partials are continuous on an open region of space containing a surface S.

$\bigtriangledown = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle$

$div(\mathbf{F}) = \bigtriangledown \bullet \mathbf{F}$

$\mathbf{curl}(\mathbf{F}) = \bigtriangledown \times \mathbf{F}$

$\oint_C \mathbf{F} \bullet \, d\mathbf{r} = \iint_S \mathbf{curl}(\mathbf{F}) \bullet \mathbf{n} \; dS,$
where C is the boundary of S and $\mathbf{n}$ is the upward unit normal on S.

$\iint_S \mathbf{F} \bullet \mathbf{n} \; dS = \iiint_Q div(\mathbf{F}) \; dV,$

where S is the closed piece-wise smooth boundary of Q and $\mathbf{n}$ is the outward unit normal on S.

If C is a piece-wise smooth simple closed plane curve ($z = 0$) bounding a plane region R, then

$\oint_C \mathbf{F} \bullet d\mathbf{r} = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) \, dA.$