SOLUTIONS TO REVIEW PROBLEMS

Chapter 10

2. $a+b=\langle 1,1\rangle$; $4b=\langle 8,12\rangle$; $\|2b-a\|=\sqrt{89}$
10. $\overrightarrow{PQ}=\langle -2,3\rangle$
12. $\frac{\mathbf{v}}{\|\mathbf{v}\|}=\frac{\langle -2,3\rangle}{\sqrt{13}}$
18. $d=\sqrt{14}$
22. $\langle 160,160\rangle+\langle -160,120\rangle+\langle 0,-300\rangle=\langle 0,-20\rangle$
30. $\cos\theta=\frac{2}{5\sqrt{5}}$; $\theta\approx 79.69^{\circ}$
32. $comp_{\langle 2,0,-3\rangle}\langle 1,3,-2\rangle=\frac{8}{\sqrt{13}}$; $proj_{\langle 2,0,-3\rangle}\langle 1,3,-2\rangle=\frac{8}{13}\langle 2,0,-3\rangle$
34. $a\times b=\langle 4,2,2\rangle$
38. $a\times b=\langle -2,-4,-5\rangle$; $u=\pm\frac{\langle -2,-4,-5\rangle}{\sqrt{45}}$
42. $d=\frac{\sqrt{174}}{\sqrt{29}}\approx 2.45$
44. $V=12$
50. $x=2t$, $y=2-3t$, $z=1-t$; $z-1=x/2=(2-y)/3$
52. $\cos\theta=\frac{-3}{\sqrt{11}}$; $\theta\approx 154.76^{\circ}$
54. Intersect at $(0,2,4)$.
58. $y=-1$

Chapter 11

14. $\pi\int_0^2\sqrt{1+16\sin^2 4\pi t}dt\approx 17.63$
20. All real numbers except $x\in\left\{\pm 1,\frac{(2k+1)\pi}{4}\right\}$, k any integer.
28. $\mathbf{v}=\left\langle 2t,0,3t^2\right\rangle$; $a=\langle 2,0,6t\rangle$
30. $\mathbf{r}(t)=\left\langle 4t-4,\frac{t^3}{3}-t+2\right\rangle$
36. Approximately 3.638 seconds to impact at $x=205.8$ ft.
Approximately 1.77 seconds to maximum height of 55.98 ft.
Impact speed $=\left\|\left\langle 40\sqrt{2},-60.487\right\rangle\right\|=82.82$ ft/sec.
42. $\kappa(t)=\frac{2\sqrt{36t^4+4t^2+9}}{t\sqrt{16t^3+9t^2+4}}$; $\kappa(0)$ is undefined; $\kappa(2)\approx 0.00569$

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20. cone
30. paraboloid
34. hyperboloid of 2 sheets
36. hyperboloid of 1 sheet
38. hyperbolic paraboloid
40. ellipsoid

Chapter 12

14. (a) 0 (b) $1-\pi$
18. Along y-axis: limit $=0$; along $y=x$: limit $=1/3$
24. $f_x=e^{xy}(xy+1)$; $f_y=x^2e^{xy}+6y$

40. $g(x,y,z) = x^2z = xy^2 + 3y - z$;
$\bigtriangledown g(x,y,z) = \left\langle 2xz - y^2, 3 - 2xy, x^2 - 1 \right\rangle\big|_{(1,-1,2)} = \langle 3,5,0\rangle$
$\langle 3,5,0\rangle \cdot \langle x-1, y+1, z-2\rangle = 0$ or $3x + 5y = -2$.

42. $\bigtriangledown f = \langle 8x, -1\rangle$; $\frac{\partial g}{\partial u} = \left\langle 8\left(u^3v + \sin u\right), -1\right\rangle \cdot \left\langle 3u^2v + \cos u, 0\right\rangle$
$\frac{\partial g}{\partial v} = \left\langle 8\left(u^3v + \sin u\right), -1\right\rangle \cdot \left\langle u^3, 8x\right\rangle$

46. $\frac{\partial z}{\partial x} = -\frac{2xz - y^2}{x^2 - 1}$; $\frac{\partial z}{\partial y} = -\frac{-2xy + 3}{x^2 - 1}$

50. $\left\langle 2x + y^2, 2xy\right\rangle\big|_{(2,1)} = \langle 5,4\rangle$; $\langle 5,4\rangle \cdot \frac{\langle 3,-2\rangle}{\sqrt{13}} = \frac{7}{\sqrt{13}}$

54. $\|\bigtriangledown f(2,1)\| = \|\langle 5,4\rangle\| = \sqrt{41}$;
max increase $= \sqrt{41}$; min decrease $= -\sqrt{41}$.

60. Critical points $(0,0)$ and $\left(\pm\frac{1}{4\sqrt{3}}, \frac{1}{12}\right)$

66. $D(0,0) = 0$ inconclusive;
$D\left(\pm\frac{1}{4\sqrt{3}}, \frac{1}{12}\right) > 0$ and $f_{xx}\left(\pm\frac{1}{4\sqrt{3}}, \frac{1}{12}\right) > 0$ imply
$f\left(\pm\frac{1}{4\sqrt{3}}, \frac{1}{12}\right)$ local mins.

68. Critical points $(0,0), \left(\pm\frac{2\sqrt{2}}{3}, \pm\frac{2\sqrt{2}}{3}\right)$; $f(0,0) = 0$;
$f\left(\pm\frac{2\sqrt{2}}{3}, -\frac{2\sqrt{2}}{3}\right) < 0$ is min; $f\left(\pm\frac{2\sqrt{2}}{3}, \frac{2\sqrt{2}}{3}\right) > 0$ is max.

Chapter 13

4. $\frac{1}{4}\left(e^2 - e^4 - e^{14} + e^{16}\right)$

8. $\frac{32}{5}$

14. $\int_0^1 \int_0^{1-x} 4 - x^2 - y^2 \; dy \; dx = \frac{11}{6}$

26. $\int_0^4 \int_0^{\sqrt{y}} f(x,y) \; dx \; dy$

28. $\int_0^{\frac{\pi}{2}} \int_0^2 2r^2 \; dr \; d\theta = \frac{8\pi}{3}$

34. $\int_0^{\frac{\pi}{4}} \int_0^{\sin 4\theta} r \; dr \; d\theta = \frac{\pi}{16}$

Set up but do not integrate:

22. $\int_0^{2\pi} \int_0^{\sqrt{3}} \int_{2+\sqrt{4-r^2}}^1 dz \; r \; dr \; d\theta$

30. $m = \int_0^2 \int_y^{4-y} 2y \; dx \; dy$

38. $SA = \int_{-2}^2 \int_{x^2}^4 \sqrt{4x^2 + 37} dy \; dx = \int_0^4 \int_{-\sqrt{y}}^{\sqrt{y}} \sqrt{4x^2 + 37} dx \; dy$

46. $\int_0^{2\pi} \int_0^2 \int_0^r 3r^2 \cos\theta \; dz \; dr \; d\theta$

54. $\int_0^{\pi} \int_0^{\frac{\pi}{2}} \int_0^2 \rho^2 \sin\phi \; d\rho \; d\phi \; d\theta = \int_0^{2\pi} \int_0^2 \int_0^{\sqrt{4-r^2}} dz \; r \; dr \; d\theta$

42. $\int_0^4 \int_0^{2-x/2} \int_0^{4-x-2y} x + y + z \; dz \; dy \; dx$
$= \int_0^2 \int_0^{4-2y} \int_0^{4-x-2y} x + y + z \; dz \; dx \; dy$

46. $\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^{\sqrt{8}} \rho^5 \sin\phi \; d\rho \; d\phi \; d\theta = \int_0^{2\pi} \int_0^2 \int_r^{\sqrt{8-r^2}} \left(r^2 + z^2\right)^{3/2} dz \; r \; dr \; d\theta$

Chapter 14

6. $\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = 0$; $f(x,y) = xy^2 + 2xe^{2y} + C$

10. $y^6 = Ae^{x^2}$ or $y = Ae^{x^2/6}$

14. $x = 4\cos t$; $y = 4\sin t$; $0 \le t \le 2\pi$; $ds = 4dt$;
$\int_C \left(x^2 + y^2\right) ds = \int_0^{\pi} 64 dt = 64\pi$

20. $\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = -2$; $W = \iint_R -2 \; dA = -8\pi$

28. $\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = 0$; $xy^2 + y - \frac{1}{3}x^2\Big|_{(3,2)}^{(1,2)} = -28\frac{1}{3}$

32. $\bigtriangledown \times F = \langle 0, 0, 0 \rangle$; $xyz - \frac{1}{2}\left(x^2 + y^2 + z^2\right)\Big|_{(2,0,0)}^{(0,1,-1)} = 1$

Set up but do not integrate:

26. $m = \int_0^{2\pi} \int_0^2 12\sqrt{1 + 4r^2} r \; dr \; d\theta$

36. $\int_0^2 \int_{x^2}^{2x} -y - 3x^2 dy \; dx$

38. $\int_0^2 \int_x^2 -x^2 dy \; dx$