CALCULUS III — EXAM I

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Name _____

Instructions: Write answers in the space provided. Show work for partial credit. Calculators may be used. Each problem is worth 5 points.

Problems 1-8 refer to the vectors $\mathbf{a} = \left\langle -2, \sqrt{3}, 3 \right\rangle$, $\mathbf{b} = \langle 1, -1, 2 \rangle$ and $\mathbf{c} = \langle 3, 0, -4 \rangle$.

1. Evaluate or simplify the following:

   a. $5\mathbf{b} - 3\mathbf{c} =$

   b. $\|\mathbf{a}\| =$

2. Find:

   a. The *cosine of the angle* θ between $\mathbf{a}$ and $\mathbf{c}$.

   b. The *angle* θ between $\mathbf{b}$ and the positive z-axis (measured in degrees).

3. Find a non-zero vector $\mathbf{w}$ *orthogonal* to $\mathbf{b}$.

   $\mathbf{w} =$

4. Find:

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5. Find the *projection of* $\mathbf{b}$ *on* $\mathbf{c}$.

   $\mathbf{proj_c b} =$

6. Find the *area* A of the parallelogram determined by $\mathbf{b}$ and $\mathbf{c}$.

   $A =$

7. Find the *volume* V of the parallelopiped determined by $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$.

   $V =$

8. Find *parametric equations* for the line passing through the point $P\,(1, 2, 5)$ and parallel to $\mathbf{b}$.

   $x =$

   $y =$

   $z =$

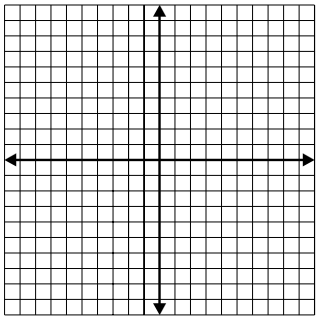
9. The xyz-coordinates of P are $(4, 3, 5)$; the xyz-coordinates of Q are $(1, 7, -5)$. Find the x, y and z *components* of the position vector $\overrightarrow{PQ}$.

10. Find the *equation* of the plane containing the points $P\,(2,1,1)$, $Q\,(3,-2,5)$, and $R\,(4,5,-2)$.

11. Find the *distance* d from the point $P\,(2,1,5)$ and the plane $5x-3y+7z=10$.

    $d=$

12. By eliminating the parameter, find the xy-equation of the graph of the parametric equations $x=t^2-\frac{1}{2}t^3$, $y=\frac{t}{2}$, $-2\le t\le 3$. Sketch the graph, indicating the (x,y)-cordinates of the endpoints and the direction of increasing parameter t.



13. Two lines are given parametrically by

$$\begin{array}{rclcl} x &=& 4-3t & ; & \qquad x=-2+3s \\ y &=& 1+2t & ; & \qquad y=-1+s \\ z &=& 3+4t & ; & \qquad z=7-2s \end{array}$$

    Determine whether or not the lines intersect. If they do, find the (x,y,z)-coordinates of their point of intersection.

14. A force of 80 lbs. is applied to a wagon handle at an angle of 45°. Find the *work* W done after pulling the wagon 300 ft.

$W =$

15. A basketball player releases a jump shot 8 feet above the floor at a 45° angle with initial speed of 22 ft/sec and sinks the shot. The rim is 10 feet above the floor. Find the *horizontal distance* x from the point of the shot to the rim.

$x =$

Problems 16-20 refer to the position function $\mathbf{r}(t) = \langle 4\sin t, t, 4\cos t \rangle$.

16. Find the *position, velocity, speed and acceleration when* $t = 0$.

a. $\mathbf{r}(0) =$

b. $\mathbf{v}(0) =$

c. $speed(0) =$

d. $\mathbf{a}(0) =$

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17. Find the *arc length* s of the curve traced out by $\mathbf{r}$ as t varies from 0 to 2π.

    $s =$

18. Find the *curvature* κ when $t = 0$.

    $\kappa =$

19. Find the unit tangent vector $\mathbf{T}$ and the principal unit normal vector $\mathbf{N}$ when $t = 0$.

    a. $\mathbf{T}(0) =$

    b. $\mathbf{N}(0) =$

20. Find the *tangential and normal components of acceleration* $a_{\mathbf{T}}$ and $a_{\mathbf{N}}$ when $t = 0$.

    a. $a_{\mathbf{T}}(0) =$