CALCULUS III — EXAM I

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Instructions: Write answers in the space provided. Show work for partial credit. Calculators may be used. Each problem is worth 5 points.

Problems 1-9 refer to the vectors $\mathbf{a} = \langle -2, 1, 2 \rangle$, $\mathbf{b} = \langle 1, -4, 3 \rangle$ and $\mathbf{c} = \langle -4, 0, 3 \rangle$.

1. Simplify: $2\mathbf{c} - 3\mathbf{a} =$

2. Find the *unit vector* $\mathbf{u}$ in the direction *opposite* from $\mathbf{c}$.

   $\mathbf{u} =$

3. Compute: $\mathbf{b} \bullet \mathbf{c} =$

4. Find the *component of* $\mathbf{a}$ *on* $\mathbf{c}$.

   $\text{comp}_{\mathbf{c}}\, \mathbf{a} =$

5. Compute: $\|\mathbf{a}\| =$

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6. Find the *cosine of the angle* θ between $\mathbf{b}$ and $\mathbf{c}$.

   $\cos\theta =$

7. Find the *volume* V of the parallelopiped determined by $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$.

   $V =$

8. Find *parametric equations* for the line passing through the point $P(3, 1, 2)$ and parallel to $\mathbf{b}$.

   $x =$

   $y =$

   $z =$

9. Find two vectors $\mathbf{w}$ parallel to $\mathbf{c}$ with norm 6.

   $\mathbf{w} =$

10. Find the *area* A of parallelogram $PQRS$ where $P(1, 2, 3)$, $Q(2, -2, 6)$, $R(4, -1, 4)$ and $S(3, 3, 1)$.

    $A =$

11. Find the equation of the plane containing parallelogram $PQRS$ where $P\,(1,2,3)$, $Q\,(2,-2,6)$, $R\,(4,-1,4)$ and $S\,(3,3,1)$.

12. Two intersecting lines are given parametrically by

$$\begin{array}{rcrcrcl} x & = & 1 & + & 2t & ; & \qquad x = 1 + s \\ y & = & 4 & - & 3t & ; & \qquad y = 4 + 3s \\ z & = & 3 & + & 4t & ; & \qquad z = 3 - 2s \end{array}$$

Find the *equation* of the plane containing them.

13. The two planes $x + y - z = 5$ and $3x - 2y + z = 7$ intersect along a line ℓ. Find *parametric equations* of the line ℓ.

$x =$

$y =$

$z =$

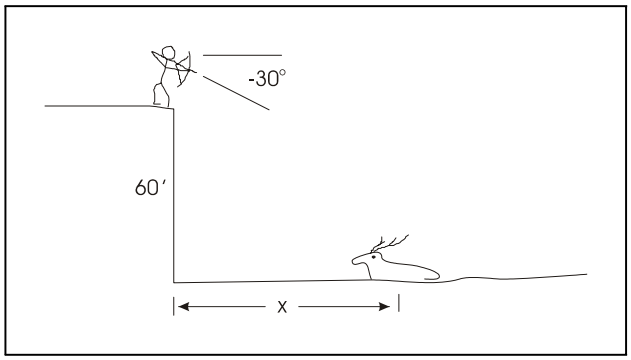
14. Find the *distance* d from the point $(3,2,4)$ to the plane given by $3x - y + z = 5$.

$d =$

15. A force of 40 lbs. is applied to a wagon handle at an angle of 30°. Find the work W done after pulling the wagon 100 ft.

$W =$

16. An archer standing on a vertical cliff spots a buck resting on the ground 60 feet below. Taking aim with an angle of depression of $-30°$ (below the horizontal), the arrow is released with an initial speed of 224 ft./sec. and kills the buck. What is the distance x from the base of the cliff to the buck?



$x =$

Problems 17-20 refer to the position function $\mathbf{r}(t) = \langle t^2, t^3 \rangle$

17. Find the *position, velocity, speed and acceleration when* $t = 1$.

a. $\mathbf{r}(1) =$

b. $\mathbf{v}(1) =$

c. $speed(1) =$

d. $\mathbf{a}(1) =$

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18. Find the *arc length* s of the curve traced out by $\mathbf{r}$ as t varies from 0 to 1.

    $s =$

19. Find the *curvature* κ when $t = 1$.

    $\kappa =$

20. Find the *tangential and normal components of acceleration* $a_{\mathbf{T}}$ and $a_{\mathbf{N}}$ when $t = 1$.

    a. $a_{\mathbf{T}}(1) =$

    b. $a_{\mathbf{N}}(1) =$