Math 311 – Calculus III – Exam 2

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Name ______________________________

Instructions. Write answers and supporting calculations in the space provided. Show work for partial credit. With the exception of problem 1, each answer is worth 5 points.

1. Match each equation in the right-hand column with the name of the surface it defines given in the left-hand column. (Each answer is worth 2 points.)

a. Cone	$4x^2 + 16y^2 - 25z = 0$ _______
b. Cylinder	$4x^2 + 16y^2 - 25z^2 = 0$ _______
c. Ellipsoid	$4x^2 + 16y^2 - 25z^2 = 36$ _______
d. Hyperbolic paraboloid	$4x^2 - 16y^2 - 25z = 0$ _______
e. Hyperboloid of one sheet	$4x^2 + 16y^2 + 25z^2 = 36$ _______
f. Hyperboloid of two sheets	
g. Paraboloid	
h. Sphere	

2. Evaluate: $\displaystyle\lim_{(x,y)\to(1,1)} \frac{x^2 - y^2}{x - y} =$

3. Show that the following limit does not exist: $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^4 + y^4}$.

5. Let $f(x, y, z) = xy^2 - y^2 z^3$; find $\frac{\partial^2 f}{\partial z \partial y} =$

6. Find the equation of the plane tangent to the graph of the function $f(x, y) = xy^2 + x^2 y^3$ at $(2, 1, 6)$.

7. The equation of the plane tangent to the graph of the function $f(x, y) = y^2 - 2x^2$ at $(2, 3, 1)$ is $8x - 6y + z = 1$. Use this equation to linearly approximate $f(1.95, 3.02)$.

8. Let $f(x, y, z) = x^3 y^2 z$. Evaluate: $\nabla f(1, 2, 3) =$

9. Find parametric equations for the line normal to the graph of $f(x, y) = xy$ at $(4, 2, 8)$.

10. Assume that the equation $2x^3 y + 3x^2 y^2 = \sin(xy)$ implicitly defines x as a function of y and find: $\frac{\partial x}{\partial} =$

11. Let $z = x^2 + y^3$, $x(r, \theta) = r\cos\theta$, and $y(u, v) = r\sin\theta$. Find $\dfrac{\partial z}{\partial \theta} =$

12. Find the maximum rate of change of $f(x, y) = 3xy^2$ at the point $P(2, -1)$.

13. Let S be the surface defined by $x^2 + 2y^2 - 5z^2 = 4$. Then S is the level surface along which $g(x, y, z) = x^2 + 2y^2 - 5z^2$ has constant value 4. Find a vector normal to S at the point $(1, 2, 1)$.

14. Let $f(x, y) = x^2 - y^3$. Find the rate of change of f in the direction of $\langle 4, 3 \rangle$ at the point $(2, -1)$.

15. Find all critical points of $f(x, y) = 3xy - x^3y + y^2$.

17. The critical points of $f(x, y) = 2x^4 - xy^2 + 2y^2$ are $(0, 0)$ and $(2, \pm 8)$. If possible, use the Second Derivative Test to determine whether a saddle point or local extremum occurs at these critical points. Determine whether each local extremum is a local maximum or local minimum.

18. Consider a rectangular box with length x, width y, height z and surface area S. Using Lagrange multipliers, find values for x, y and z in terms of P that maximize volume. (Solutions without work receive no credit.)

19. Find the absolute maximum and minimum values of the function $f(x, y) = 2xy$ subject to the constraint $x^2 + y^2 = 4$.