CALCULUS III — EXAM II

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Name ______________________________

Instructions: Write answers and supporting calculations in the space provided. Show work for partial credit. Each answer is worth 5 points.

1. Match each equation in the right-hand column with the name of the surface it defines given in the left-hand column. (Each answer is worth 1 point.)

a. Cone	$2x^2 - 3y^2 - 6z = 0$ ______
b. Cylinder	$2x^2 + 3y^2 - 6z = 0$ ______
c. Ellipsoid	$2x^2 - 3y^2 - 6z^2 = 6$ ______
d. Hyperbolic paraboloid	$2x^2 + 3y^2 - 6z^2 = 6$ ______
e. Hyperboloid of one sheet	$2x^2 + 3y^2 + 6z^2 = 6$ ______
f. Hyperboloid of two sheets	
g. Paraboloid	
h. Sphere	

2. Is the function

$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

continuous or discontinuous at $(0,0)$? Explain.

3. Show that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3y}{x^4+y^4}$ does *not* exist.

4. Given the fact that the following limit exists, evaluate:

$$\lim_{(x,y)\to(0,0)} \frac{\sin(x+y)}{x+y} =$$

5. The vertical plane $y = 2$ is parallel to the xz-coordinate plane and intersects the graph of $z = 2xy^2$ along some curve C. Find the slope of the line tangent to C at the point $(1, 1, 2)$.

6. Evaluate the gradient of the function $f(x, y) = 3x^2 - 12xy + 2y^3$ at the point $(1, -1)$.

7. Find $\frac{\partial z}{\partial v}$, if $z = xy$, $x(u, v) = v + \cos u$ and $y(u, v) = u + \sin v$.

   $\frac{\partial z}{\partial v} =$

8. The equation $yz^2 + x^3z = 0$ implicitly defines z as a function of x and y.

   Find $\frac{\partial z}{\partial x} =$

9. Find the equation of the plane tangent to the graph of $g(x, y) = 3x^2 - 12xy$ at $(1, 0, 3)$.

10. Find the linear (or tangent) approximation for $f(x, y) = \sqrt{4x + 2y}$ at the point $(3, 2)$ and use it to estimate $f(3.1, 1.9)$.

11. Find the rate of change of $f(x, y) = x^2 - y^2$ at the point $P(-1, 3)$ in the direction of $\langle 4, -3 \rangle$.

12. Find a non-zero vector in the direction of the maximum rate of change of $g(x, y) = xy$ at the point $P(1, -2)$.

13. Find the maximum rate of change of $g(x, y) = xy$ at the point $P(1, -2)$.

14. Find parametric equations for the line normal to the graph of $z = x^2 - y^2$ at the point $(3, 2, 5)$.

15. Assume y is implicitly defined as a function of x by the equation $xy^2 + x^2y = \sin x$. Use methods of Calculus 3 to find $\frac{dy}{dx}$.

$\frac{dy}{dx} =$

16. Think of the surface defined by $x^2 + 3y^2 - 2z^2 = 4$ as the level surface S of height 4 for the function $g(x, y, z) = x^2 + 3y^2 - 2z^2$. Find the equation of the tangent plane to S at the point $(3, 1, 2)$.

17. Find all critical points of $h(x, y) = 3x - x^3 + y^2 x - 2y^2$.

18. The critical points for $f(x, y) = 3x^2 - 12xy + 2y^3$ are $(0, 0)$ and $(4, 8)$. If any, find all saddle points on the graph of f and/or local extreme values of f.

19. Use Lagrange multipliers to find the absolute extrema of $h(x, y) = x^2 + y^2$ along the line $2x + 6y = 20$.

20. Evaluate: $\int_0^3 \int_0^{\sqrt{y}} 6xy \, dx \, dy$