CALCULUS III — EXAM 3

December 1, 2006

Ron Umble, Instr.
Millersville University

Name ______________________________

Instructions: Write answers and supporting calculations in the space provided. The blank cover page may be torn off and used as scratch. Cell phones must be turned off and stowed. Unless specified otherwise, each problem is worth 10 points.

1. Evaluate: $\int_0^{\frac{\pi}{6}} \int_0^{\sin\theta} 16r\cos\theta \; dr \; d\theta =$

2. Insert the limits of integration that reverse the order of integration; <u>DO NOT EVALUATE</u>:
$\int_0^1 \int_{2x^2}^{2\sqrt{x}} \; dy \; dx.$

$$\int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} dx \; dy.$$

3. Set up but <u>DO NOT EVALUATE</u> a double integral in polar coordinates to compute the *area of the region* R outside the circle $x^2+y^2=1$ and inside the cardioid $r = 2-2\sin\theta$.

4. Set up but <u>DO NOT EVALUATE</u> a double integral to compute the *volume* of the region under the graph of $z = 4 - y^2$ and above the triangle in the xy-plane with vertices $(0,0,0)$, $(2,0,0)$ and $(0,2,0)$.

5. Transform the following integral to an integral in *rectangular coordinates* but <u>DO NOT EVALUATE</u> it:

$$\int_0^{\pi/4} \int_0^{2\sec\theta} r^2 dr d\theta$$

6. A thin circular metallic plate with density $\rho(x, y) = x^2 + y^2$ is modelled by the half-disk $R = \left\{(x, y) \,|0 \leq y = \sqrt{9 - x^2}\right\}$. Set up but <u>DO NOT EVALUATE</u> a double integral in polar coordinates to compute the moment of R about the y-axis.

7. Let Q be the space region under the graph of $z = xy$ and above the rectangle $0 \leq x \leq 2$ and $2 \leq y \leq 4$ in the xy-plane. Evaluate $\iiint\limits_Q 9z \; dzdydx$.

8. Let Q be the solid bounded by the two paraboloids $z = x^2+y^2$ and $z = 16-x^2-y^2$. Set up but <u>DO NOT EVALUATE</u> a triple integral in cylindrical coordinates to evaluate $\iiint\limits_Q x^2 + y^2 \; dzdydx$.

9. Let Q be the solid region above the cone $z = -\sqrt{x^2+y^2}$ (note the sign!) and inside the sphere$x^2 + y^2 + z^2 = 16$. Set up but <u>DO NOT EVALUATE</u> a triple integral in spherical coordinates to evaluate:

$$\iiint\limits_Q \sqrt{x^2+y^2+z^2} dV =$$

10. Find cylindrical and spherical coordinates of the point given by $\left(\sqrt{2}, \sqrt{6}, 2\sqrt{2}\right)$ in rectangular coordinates.

$r =$ _______________ $\theta =$ _______________ $z =$ _______________

$\rho =$ _______________ $\phi =$ _______________ $\theta =$ _______________

11. A thin wire wrapped around a steel bar has the shape of the helix C parametrized by $x = 2\cos t,\;\; y = 2\sin t,\; z = \sqrt{5}t,\; 0 \le t \le 100\pi$. The density ρ of the wire at (x, y, z) is given by $\rho(x, y, z) = \sqrt{5}z$. Compute the *mass* of the wire. (15 points)

12. Let S be that portion of the graph of $z = xy$ inside the cylinder $x^2 + y^2 = 4$. Set up but <u>DO NOT EVALUATE</u> a double integral in polar coordinates to compute the *surface area* of S.

13. Given that the vector field $\mathbf{F}(x, y) = \langle y \sin xy + 1, x \sin xy - 1 \rangle$ is conservative, find a potential function f for $\mathbf{F}$.

14. Let $\mathbf{F}(x, y) = \langle y, x \rangle$. Find equations in x and y for the *flow lines* of $\mathbf{F}$.

15. Compute the *work done* by the force field $\mathbf{F}(x, y) = \langle x + y, x - y \rangle$ as it moves a particle along the graph of $y = x$ from $(0, 0)$ to $(3, 3)$.

16. Use the Fundamental Theorem for Line Integrals to evaluate:

$$\int_{(1,0)}^{(2,1)} \left(y^2 + 3x^2\right) dx + (2xy - 1)\, dy.$$

(15 points)

17. Let C be the ellipse $9x^2 + 4y^2 = 36$ with positive orientation. Given that the area of the region enclosed by this ellipse is 6π, use *Green's Theorem* to evaluate $\oint_C (x - 2y)\, dx + (2x + y)\, dy$.

18. Let R be region enclosed by the ellipse $9x^2 + 4y^2 = 36$. Use *Green's Theorem* to express $\iint\limits_R dA$ as a line integral. Evaluate this line integral and obtain the area of R.

19. Let $\mathbf{F}(x, y, z) = \langle xe^z, ye^x, ze^y \rangle$.