CALCULUS III — EXAM III

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Name ____________________________

Instructions: Write answers and supporting calculations in the space provided. Unless specified otherwise, each problem is worth 5 points.

1. Consider a thin rectangular metallic plate $R = [0,1] \times [0,2]$ with density $\rho(x,y) = 2y$. Compute the *moment* M_x about the x-axis.

2. Reverse the order of integration: $\int_0^4 \int_{-\sqrt{4-x}}^{\sqrt{4-x}} dy\, dx$. <u>DO NOT EVALUATE</u>.

$$\int_{\Box}^{\Box} \int_{\Box}^{\Box} dx\, dy.$$

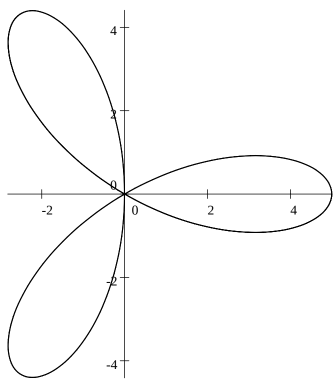
3. Set up a double integral to compute the *area* of the region R bounded by the curves $y = x^3 - x$ and $y = x - x^3$. <u>DO NOT EVALUATE</u>.

4. Set up a double integral to compute the *volume* of the solid bounded by the graphs of $y = 0$, $y = 9 - x^2$, $z = 0$ and $z = 9 - x^2$. <u>DO NOT EVALUATE</u>.

5. Transform to *polar coordinates*: $\int_0^{\sqrt{2}} \int_x^{\sqrt{4-x^2}} \sqrt{x^2+y^2}\, dy\, dx$

DO NOT EVALUATE.

6. Set up a double integral in polar coordinates whose value is the *area* enclosed by one petal of the three-leaf rose $r = 5\cos 3\theta$. DO NOT EVALUATE.



7. Set up a double integral in cylindrical coordinates to compute the *surface area* of the paraboloid $z = 25 - x^2 - y^2$ with $z \geq 0$ (above the xy-plane). DO NOT EVALUATE.

8. Let Q be the region of space under the graph of $z = x^2 + 2y + 4$ and above the rectangle $[0,1] \times [0,3]$ in the xy-plane. Evaluate $\iiint_Q dz dy dx$.

9. Let Q be the solid in the first octant bounded the coordinates planes, the plane $z = 5$ and the cone $z = \sqrt{x^2 + y^2}$. The density of Q is $\rho(x, y, z) = 2xy$. Set up a triple integral in *cylindrical coordinates* to compute the mass of Q. <u>DO NOT EVALUATE</u>.

10. Let Q be the region of space inside the hemisphere $z = \sqrt{4 - x^2 - y^2}$, outside the hemisphere $z = \sqrt{1 - x^2 - y^2}$ and inside the cone $z = \sqrt{x^2 + y^2}$. Set up a triple integral in *spherical coordinates* to evaluate $\iiint\limits_Q x^2 + y^2 + z^2 \, dV$. <u>DO NOT EVALUATE</u>.

11. Find *rectangluar* and *cylindrical* coordinates of the point $\left(2, \frac{\pi}{6}, \frac{2\pi}{3}\right)$ in spherical coordinates.

$r =$ ________________ $\theta =$ ________________ $z =$ ________________

$x =$ ________________ $y =$ ________________ $z =$ ________________

12. Let $\mathbf{F}(x, y) = \langle -y, x \rangle$. Find equations in x and y for the *flow lines* of $\mathbf{F}$.

13. Let C be the curve parametrized by $x = t^2$, $y = t^3$ with $x \in [0, 2]$. Evaluate

$$\int_C y\ dx + x\ dy.$$

14. Compute the *work* done as a force $\mathbf{F}(x, y) = \langle y, x\rangle$ moves a particle along the line segment from $(0, 0)$ to $(3, 4)$ parametrized by $x = 3t$ and $y = 4t$ with $0 \le t \le 1$.

15. Determine whether or not the vector field $\mathbf{F}(x, y) = \langle y \sin xy + 1, x \sin xy - 1\rangle$ is conservative.

16. Given that the vector field $\mathbf{F}(x, y) = \langle 2x + 2y^2, 3y^2 + 4xy\rangle$ is conservative, find a *potential function* for $\mathbf{F}$.

17. Given that the vector field $\langle y, x\rangle$ is conservative, use the *Fundamental Theorem of Line Integrals* to evaluate:

$$\int_{(1,0)}^{(2,1)} y dx + x dy.$$

18. Let C be the circle $x^2 + y^2 = 9$ with positive orientation. Use *Green's Theorem* to evaluate $\oint_C (x - y)\, dx + (x + y)\, dy$.

19. Let R be region enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Use *Green's Theorem* to express $\iint\limits_R dA$ as a line integral. Evaluate the line integral and obtain a formula for the area of R.

20. Let $\mathbf{F}(x, y, z) = \langle xz, xy, yz\rangle$.

    (a) $\mathbf{Curl}\,(\mathbf{F}) =$

    (b) $div\,(\mathbf{F}) =$