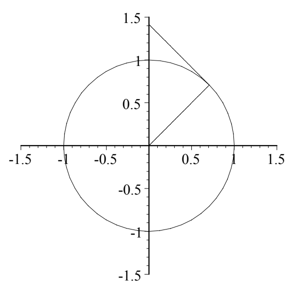
MATH 261 – CALCULUS III

THE UNIT NORMAL FOR PLANE CURVES

Given a smooth position function $\mathbf{r}(t) = \langle x(t), y(t) \rangle$, the unit normal $\mathbf{N}(t)$ can be obtained directly from the unit tangent $\mathbf{T}(t)$ by rotating $\mathbf{T}$ $90°$ to the <u>concave side</u> of the curve taced out by $\mathbf{r}$.

Example 1: When the unit cricle is traced out with counterclockwise orientation, the unit normal is obtained by rotating the unit tangent counterclockwise $90°$.

$\mathbf{r}(t) = \langle \cos t, \sin t \rangle$
$\mathbf{T}(t) = \mathbf{v}(t) = \langle -\sin t, \cos t \rangle$
$\mathbf{N}(t) = \mathbf{T}'(t) = \langle -\cos t, -\sin t \rangle$



Example 2: When the unit cricle is traced out with clockwise orientation, the unit normal is obtained by rotating the unit tangent clockwise $90°$.

$\mathbf{r}(t) = \langle \cos t, -\sin t \rangle$
$\mathbf{T}(t) = \mathbf{v}(t) = \langle -\sin t, -\cos t \rangle$
$\mathbf{N}(t) = \mathbf{T}'(t) = \langle -\cos t, \sin t \rangle$