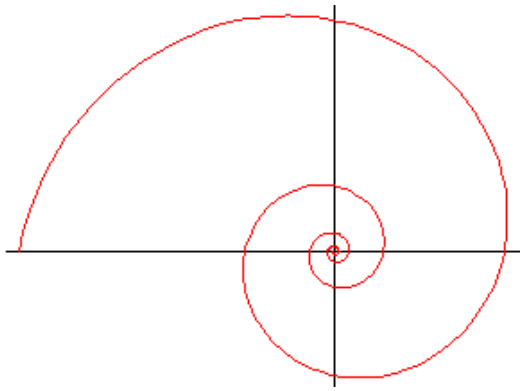
Sea Shell Spirals
by Ivars Peterson

The chambered nautilus is a sea creature that belongs in the same class as the octopus. Unlike the octopus, it has a hard shell that's divided into chambers. As the nautilus matures and grows, it periodically seals off the shell behind it and creates a new, larger living chamber. The shells of adults may have as many as 30 such chambers. This growth process yields an elegant spiral structure, visible when the shell is sliced to reveal the individual chambers. Many accounts describe this pattern as a logarithmic (or equiangular) spiral and link it to a number known as the golden ratio.

This cutaway of a nautilus shell shows its chambers and reveals an elegant spiral structure.

A logarithmic spiral follows the rule that, for a given rotation angle (such as one revolution), the distance from the pole (spiral origin) is multiplied by a fixed amount.



A logarithmic spirial.

When this fixed amount is the golden ratio, (1 + sqrt[5])/2, or 1.6180339887. . . , you get a particular type of logarithmic spiral. Such a logarithmic spiral can be inscribed in a rectangle whose sides have lengths defined by the golden ratio.

Does the spiral of a chambered nautilus shell actually fit such a model? Retired mathematician Clement Falbo contends that the nautilus shell does not have a spiral shape based on the golden ratio. In 1999, when Falbo measured nautilus shells in a collection at the California Academy of Sciences in San Francisco, he found that the spirals of these shells

could be inscribed within rectangles with sides in the ratio of about 1.33; not 1.618 . . . , as they would be if a spiral based on the golden ratio matched the shell shape. Roughly speaking, the spiral of the chambered nautilus triples in radius with each full turn whereas a golden-ratio spiral grows by a factor of about 6.85 per full turn. The measured ratios ranged from 1.24 to 1.43. "It seems highly unlikely that there exists any nautilus shell that is within 2 percent of the golden ratio, and even if one were to be found, I think it would be rare rather than typical," Falbo concludes.

In a 2002 article in the online Nexus Network Journal (see http://www.nexusjournal.com/Sharp_v4n1-pt04.html), John Sharp pointed out the same problem. Starting with the observation that shell spirals are logarithmic spirals, many people automatically assume that, because the golden ratio can be used to draw a logarithmic spiral, all shell spirals are related to the golden ratio, when, in fact, they are not. Sharp's own measurements of nautilus shells also confirmed that the golden ratio rectangular spiral and the nautilus spiral "simply do not match." Nonetheless, many accounts still insist that a cross section of nautilus shell shows a growth pattern of chambers governed by the golden ratio. "One of the amazing things about such misconceptions is that it is so widespread, even by mathematicians who should know better," Sharp observes. "It is a prime example of why geometry needs to be taught more widely and not only geometry, but the visual appreciation of shape and proportion." And it's always useful to check things out in the real world.