4-2-2013. Binomial probability and Expectation

- Binomial probability
  - A **binomial experiment** has _____ possible outcomes: _____
  - A repeated performance of a binomial experiment is a _____
  - A **random variable** *x* _____
  - A **probability distribution** lists all possible values of a _____ and their _____
  - The **sum of the probabilities** in any probability distribution is always _____
    - **Example.** Construct the binomial probability distribution for tossing 5 coins.

  - **Binomial Probability Formula**
    n *= number of independent repetitions in a Bernoulli trial*
    P *= probability of success of each repetition*
    q *= probability of failure of each repetition (q = 1 - p)*
    x *= number of successes that occur*
    n-x *= number of failures that occur*

$$P(x) = {}_nC_x \cdot p^x \cdot q^{n-x}$$

    - **Example 1**. Toss a coin five times. Find the probability of obtaining exactly three heads.

- **Example 2.** Roll a die six times. Find the probability of obtaining exactly two 5s.

- **Example 3**. A couple plans to have 5 children. Find the probability they will have more than three girls.

- **Example 4**. Andrew has a well-established career batting average of .300. If Andrew bats 10 times, find the probability that he will get more than two hits.

- **Expected value**

  - If a random variable x assumes the values $x_1, x_2, \ldots, x_n$ with probabilities $p_1, p_2, \ldots, p_n$, the **expected value of *x*** is

    $E(x) =$ _____

    - **Example 1**. A family plans to have three children. Find the expected number of boys.

    - **Example 2**. A player pays $3 to play the following game: Three coins are tossed. The game pays $1 if no heads appear, $2 if one head comes up, $3 if two heads appear, and $4 if three heads appear. Find the player's expected net winnings.

    - **Example 3**. How much should the player pay to play the game in Example 2 to have an even chance of recovering his/her money (this makes it a fair game)?

- **Example 4**. In a certain state lottery, a player plays one dollar to play the Daily Number game: The player chooses three digits from {0,1,…,9} in a specific order (000, 001, and 012 are allowed). The lottery operator randomly selects three digits. If the player's number matches, the player wins $600; otherwise he/she loses. Find the expected net winnings.

- **Example 5**. A roulette wheel has 18 black, 18 red, and 2 white compartments. The player places a $1 bet on black or red. If the player's color comes up he/she wins $2; otherwise the player loses the bet. Find the expected net winnings.