# Mathematical Research Experiences for Undergraduates at Millersville University

Ron Umble
Millersville University of PA

Hesston College Science and Math Symposium

September 24, 2010

# Three Found Problems:

- **Problem 1.** *My friend Tom Banchoff has two daughters Ann and Mary Lynn. Tom was 60 when ML turned 30. A few years later, Ann remarked that Tom was twice as old as ML again! How old were Tom and ML the <u>second</u> time?*

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- **Problem 2.** *Why do people living on the equator have 12 hours of daylight and 12 hours of darkness year round?*

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- **Problem 2.** *Why do people living on the equator have 12 hours of daylight and 12 hours of darkness year round?*

- **Problem 3.** *A traffic light is red for 45 seconds and green for 15. What is the average wait time?*

# Research Problem Solved by Robert Painter, Ellen Panofsky, Joel Mohler, and Heather Hair:

*Find all paths of minimal length connecting two points on a tin can.*

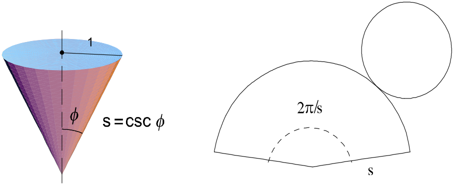
- Project proposed and codirected by Frank Morgan

<u>**"Geodesics on a Tin Can"**</u>

# Ellen wins a Mathfest 2001 award for her outstanding presentation

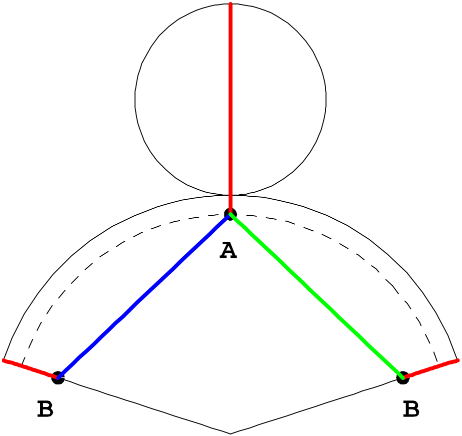
# Joel extended the solution to conical cups with lid

- Published in Pi Mu Epsilon Journal, spring 2008




Flat model of a conical drinking cup with lid.

# Three paths of minimal length from A to B

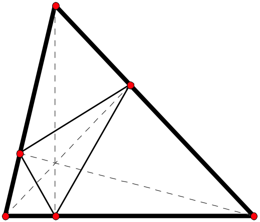


# Joel's senior thesis wins EPADEL's 2003 student paper competition

# Research Problem Studied by Andrew Baxter, John Gemmer, Kyle Gerhart, Sean Laverty, and Steve Weaver

*Find and describe the periodic trajectories (orbits) of a billiard ball in motion on a triangular air hockey table.*

- Project proposed and codirected by Dennis DeTurck



Altitudes of an acute triangle bisect interior angles of the inscribed orthic triangle

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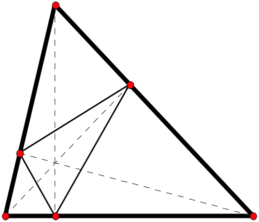
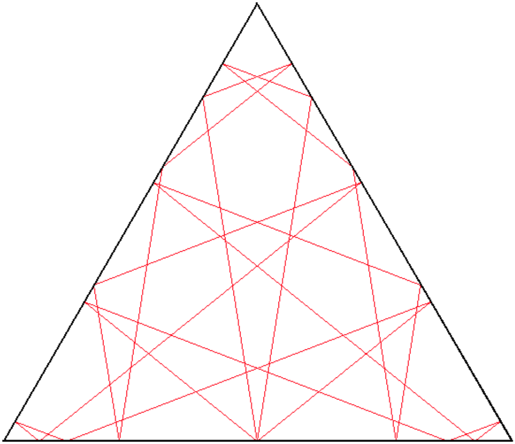


Altitudes of an acute triangle bisect
interior angles of the inscribed orthic triangle

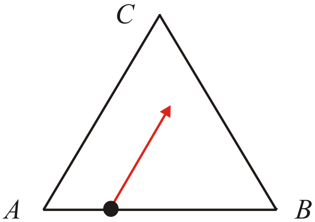
- On certain obtuse triangles, the existence of periodic orbits is unknown. This is a active area of advanced research.

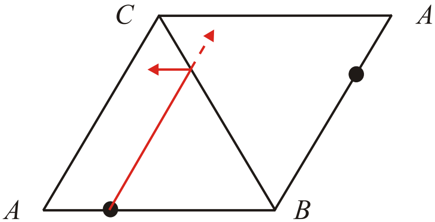
# Andrew found a complete solution for equilaterals

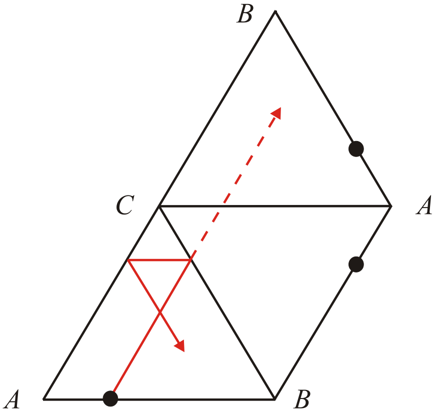
- Published in American Mathematical Monthly, June-July 2008



A periodic orbit on an equilateral triangle can be unfolded by reflecting the triangle in its edges...




C
A
A
B


B
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A
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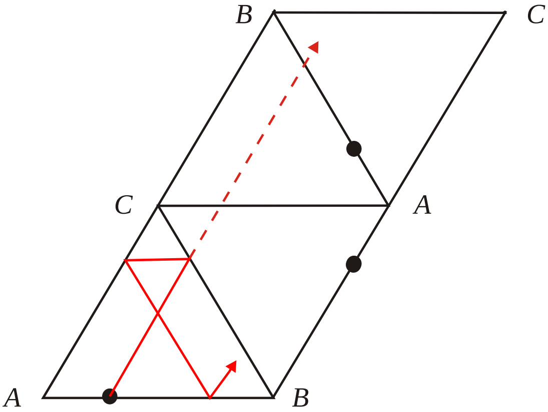
B
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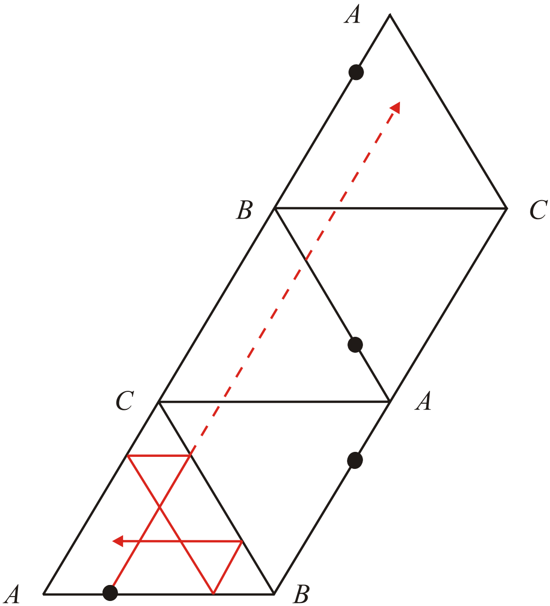
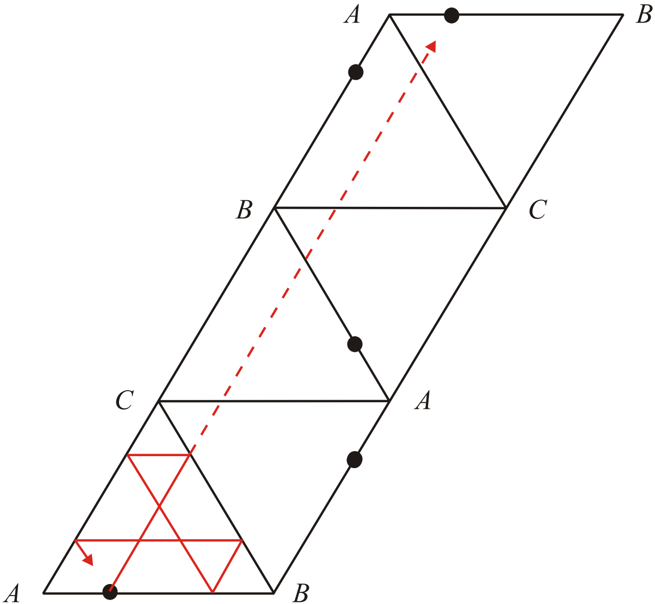

A
B
C
C
A
A
B


A
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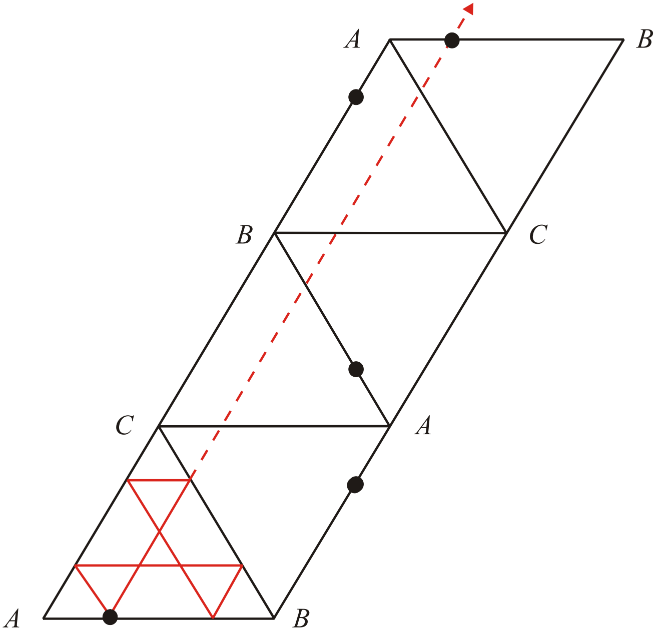
A
B
B
C
C
A
A
B

# Software by Stephen Weaver

"**Orbit Tracer 2**"

# Theorem

- *There are infinitely many distinct periodic orbits on an equilateral.*

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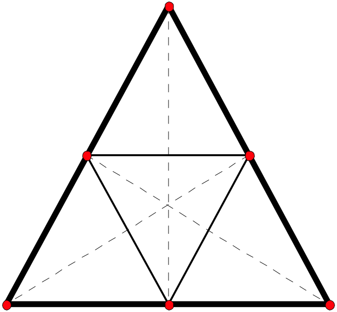
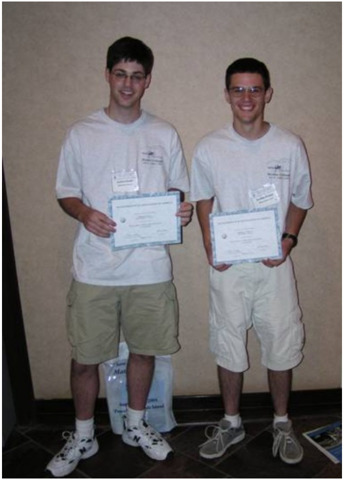
- *The only orbits of odd period are odd multiples of the orthic orbit:*



# Andrew and Steve win a Mathfest 2003 award for their outstanding presentation



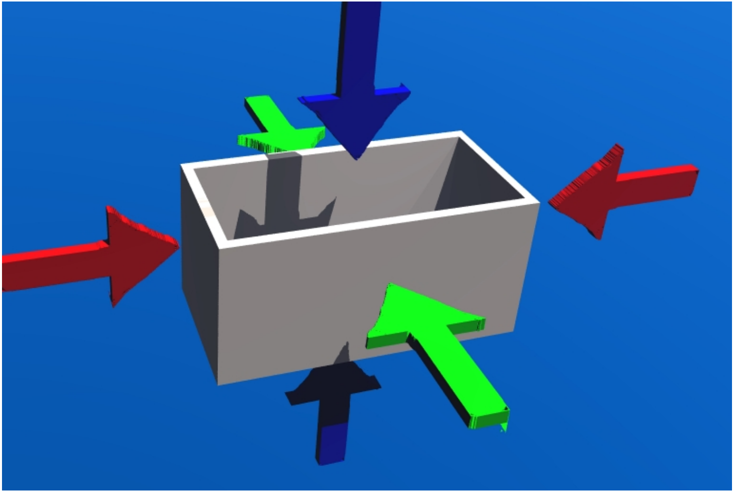
# Andrew's senior thesis wins EPADEL's 2005 student paper competition

# Research Problem Studied by Nick Brubaker, Steve Carter, Sean Evans, Dan Kravatz, Sherry Linn, Steve Peurifoy, and Ryan Walker:

*Find surface-area minimizing double bubble configurations in 3-dim'l torus*

- Project proposed and codirected by Frank Morgan

# A model of the 3-dimensional torus



# Theorem

*There are at least ten surface-area minimizing double bubble configurations in a 3-dim'l torus.*

- Paper written by MU undergraduates and published in Math Horizons, April 2008:

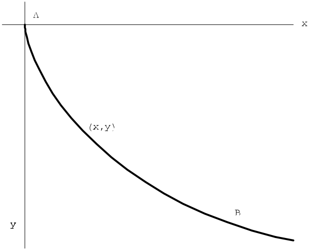
"**Double Bubble Experiments in the 3-Torus**"

# Dan Kravatz presenting our results at Mathfest 2007

# Research Problem Solved by John Gemmer:

**(Generalized Brachistochrone)** *Find a path on a smooth surface along which a bead slides from one point another in minimal time.*

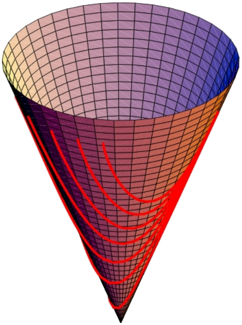
- Project proposed and codirected by Michael Nolan.

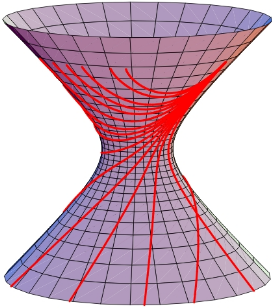



A cycloid: Newton's solution in the plane

**Animated cycloid** (software by Robert Painter)

# John's solutions on a cone and hyperboloid





# John presenting at Mathfest 2005

# John's senior thesis is co-winner of SIAM's 2005 award for best student paper in applied mathematics

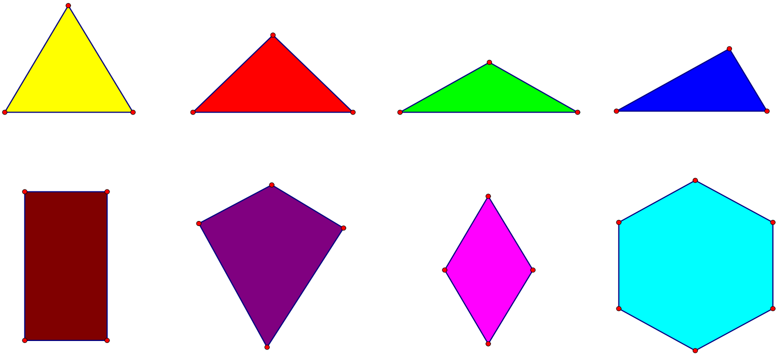
- To appear in Pi Mu Epsilon Journal, 2010.

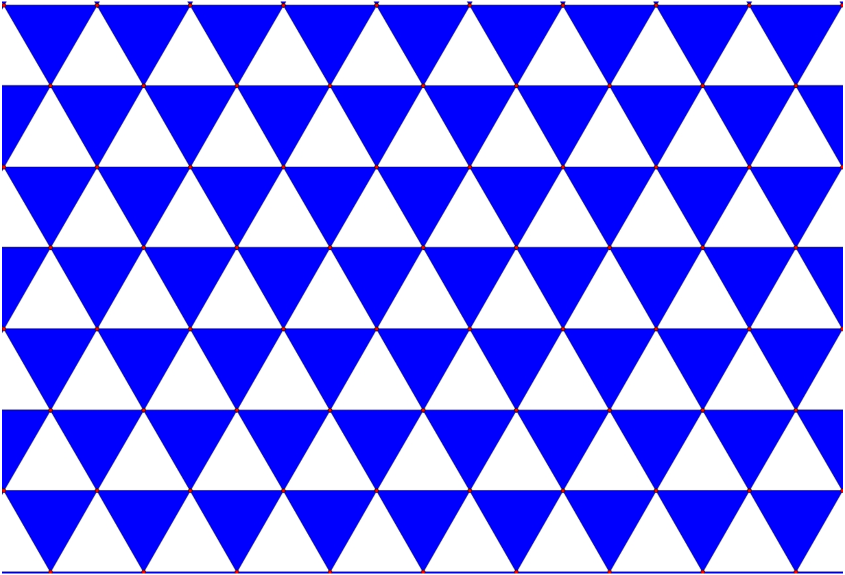
# Research Problem Solved by Matt Kirby, Josh York, and Andrew Hall:

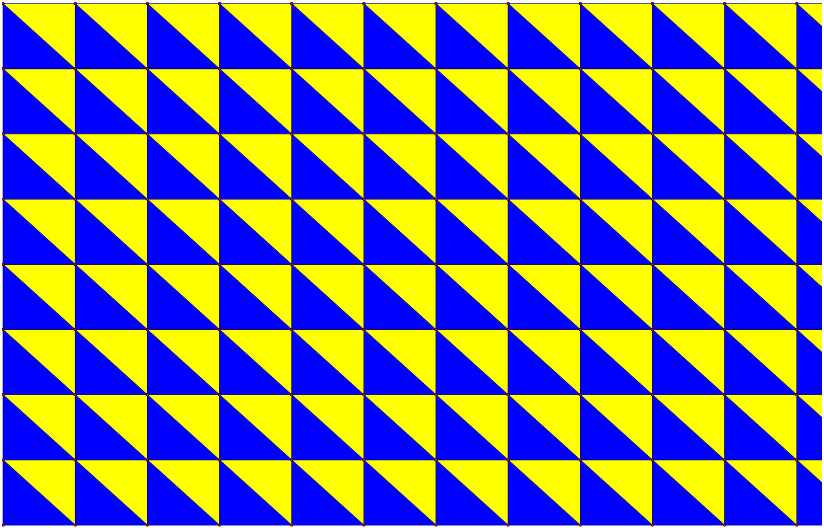
*How many ways can one configure the perforation lines on a page of postage stamps so the page folds neatly into a stack of single stamps?*

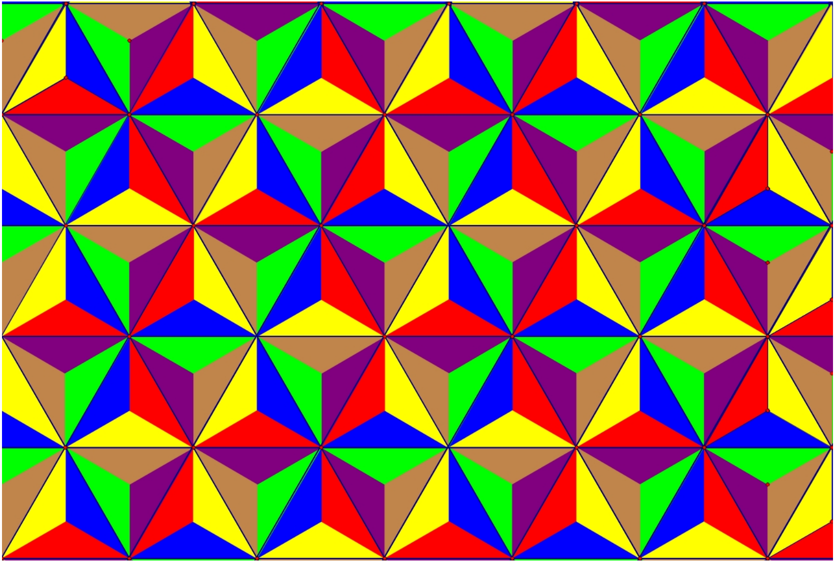
# Theorem

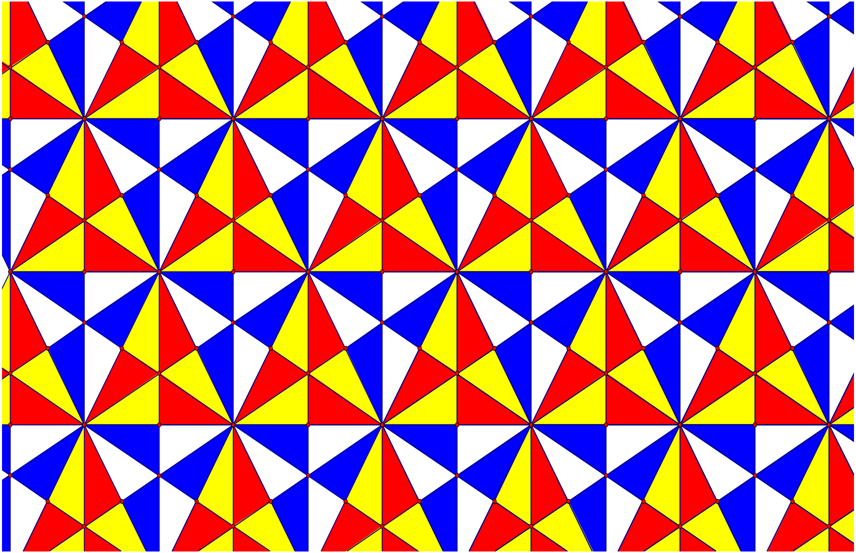
*Exactly eight polygons generate edge tessellations of the plane:*

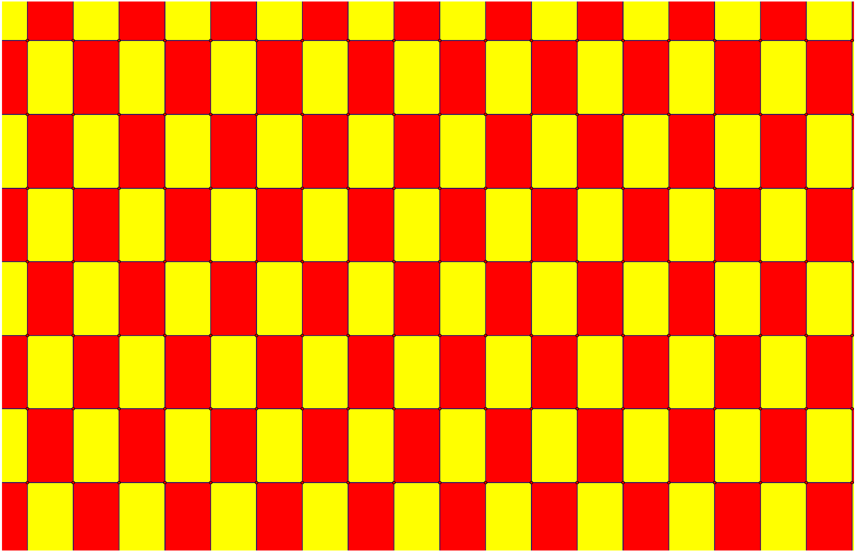


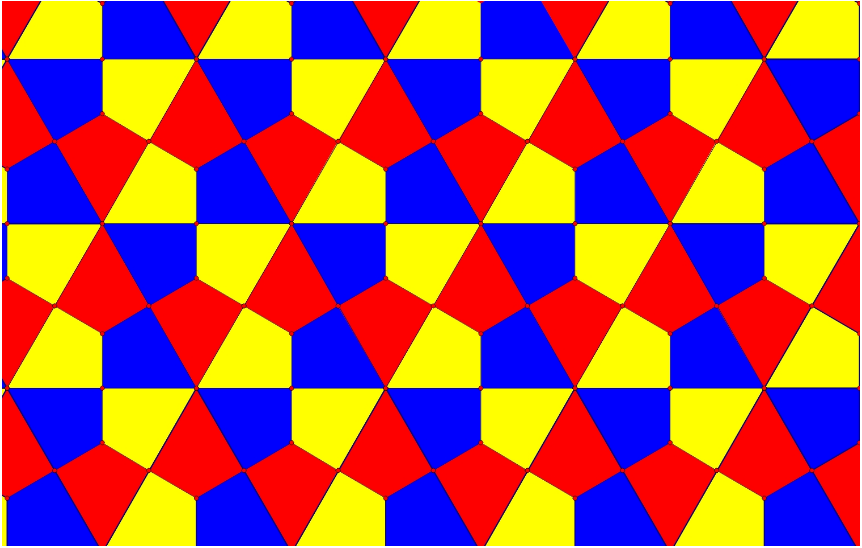


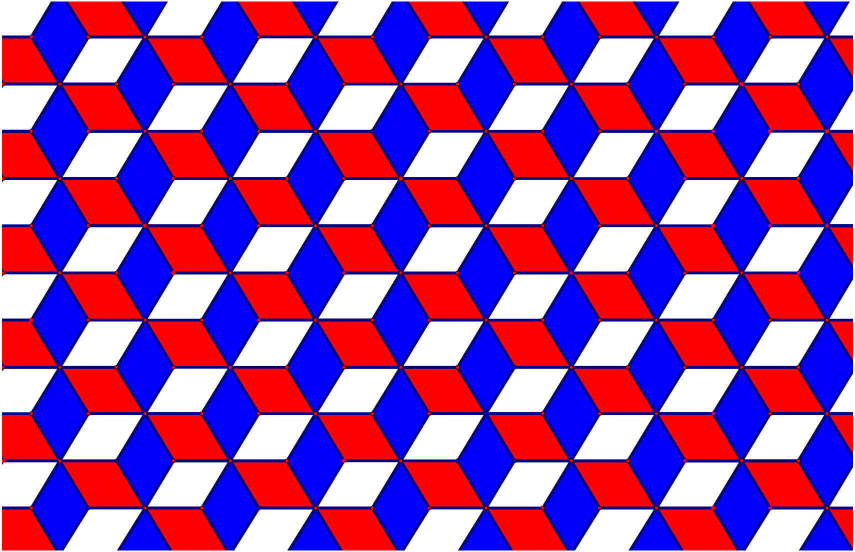


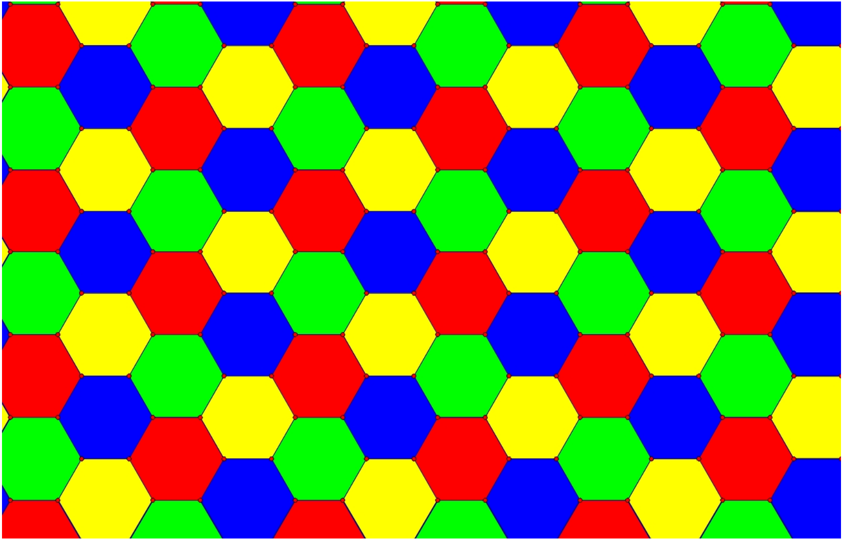






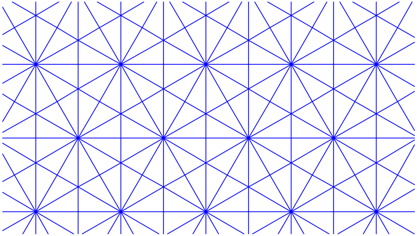


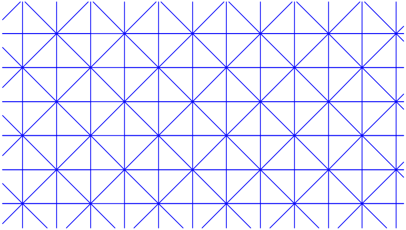


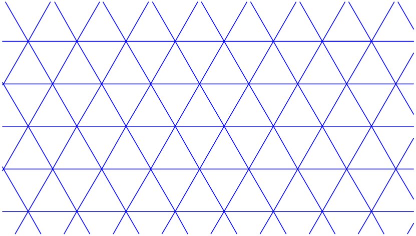


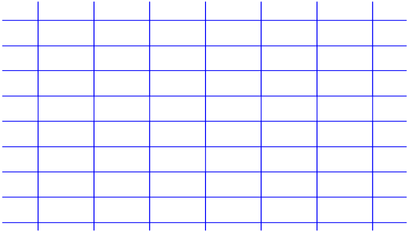
# Four non-obtuse polygons solve Stamp Folding Problem:

- Paper submitted and under review









# Open Problems for Undergraduates

- Find all paths of minimal length connecting two points on a "top hat" and a "witch's hat". Find a general procedure for computing paths of minimal length connecting two points on piece-wise smooth surfaces with singularities along smooth curves.

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- Having described the periodic orbits on an equilateral triangle, do this for each of the seven other polygons above.

- Describe the periodic orbits on an isosceles triangle.

"**Orbit Tracer 1**" (by Steve Weaver)

# Found Problem #1 Solution

- Mary Lynn turned 30 while Tom was 60.

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- *Tom turned 62 while ML was 31!*

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- Tom turned 61 while ML was 30.
- ML turned 31 while Tom was 61.
- *Tom turned 62 while ML was 31!*

- **Remark:** Unless two persons have the same birthday, the older will be twice as old as the younger *twice*.

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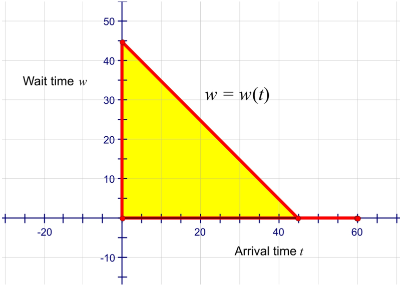
- The great circle separating day and night intersects the equator at diametrically opposite points.
- *Thus at each instant, exactly half of the equator is light and half is dark.*

**Animation of rotating earth**

# Found Problem #3 Solution:

- Given an arrival time $t$, the wait time $w = w(t)$ is



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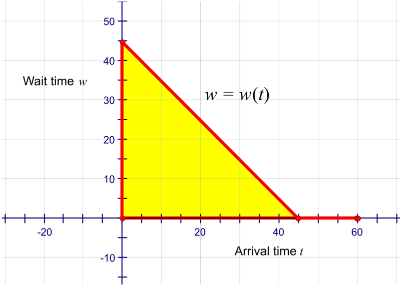


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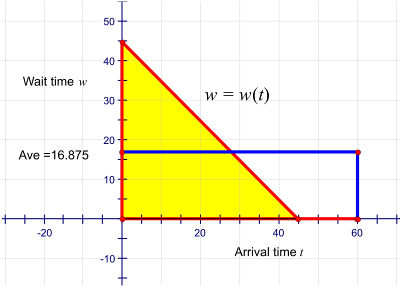


- Average wait time is the height $h$ of the rectangle with length 60 and area $45^2/2 = 1012\frac{1}{2}$.
- $h = 1012\frac{1}{2} \div 60 = 16\frac{7}{8}$ *seconds!*

# The End

HAPPY RESEARCHING!