Fall 2017 PHYS 131
Week 10 Recitation:
Chapter 10: 5, 11, 17, 27, 51.

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5. **ssm** A person who weighs 670 N steps onto a spring scale in the bathroom, and the spring compresses by 0.79 cm. **(a)** What is the spring constant? **(b)** What is the weight of another person who compresses the spring by 0.34 cm?

REASONING The weight of the person causes the spring in the scale to compress. The amount x of compression, according to Equation 10.1, depends on the magnitude F_x^{Applied} of the applied force and the spring constant k.

SOLUTION

a. Since the applied force is equal to the person's weight, the spring constant is

$$k = \frac{F_x^{\text{Applied}}}{x} = \frac{670\ \text{N}}{0.79 \times 10^{-2}\ \text{m}} = \boxed{8.5 \times 10^{4}\ \text{N/m}} \qquad (10.1)$$

b. When another person steps on the scale, it compresses by 0.34 cm. The weight (or applied force) that this person exerts on the scale is

$$F_x^{\text{Applied}} = k\,x = \left(8.5 \times 10^{4}\ \text{N/m}\right)\left(0.34 \times 10^{-2}\ \text{m}\right) = \boxed{290\ \text{N}} \qquad (10.1)$$

Note: The part (b) of problem 2 in practice exam can be solved as followings (as we did in the class):

All forces upward –All forces downward = 0

If we assume the force applied by the bone is upward with magnitude = F

We get,

$$M + F - W_{\text{ball}} - W_{\text{forearm}} = 0$$

Therefore $F = W_{\text{ball}} + W_{\text{forearm}} - M = -1010\ \text{N}$

(As M is obtained in part (a), W_{ball} and W_{forearm} are given in the problem)

That means that F= 1010 N downward

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***11.** **ssm** A small ball is attached to one end of a spring that has an unstrained length of 0.200 m. The spring is held by the other end, and the ball is whirled around in a horizontal circle at a speed of 3.00 m/s. The spring remains nearly parallel to the ground during the motion and is observed to stretch by 0.010 m. By how much would the spring stretch if it were attached to the ceiling and the ball allowed to hang straight down, motionless?

REASONING When the ball is whirled in a horizontal circle of radius *r* at speed *v*, the centripetal force is provided by the restoring force of the spring. From Equation 5.3, the magnitude of the centripetal force is mv^2/r, while the magnitude of the restoring force is kx (see Equation 10.2). Thus,

$$\frac{mv^2}{r} = kx \tag{1}$$

The radius of the circle is equal to ($L_0 + \Delta L$), where L_0 is the unstretched length of the spring and ΔL is the amount that the spring stretches. Equation (1) becomes

$$\frac{mv^2}{L_0 + \Delta L} = k\Delta L \tag{1'}$$

If the spring were attached to the ceiling and the ball were allowed to hang straight down, motionless, the net force must be zero: $mg - kx = 0$, where $-kx$ is the restoring force of the spring. If we let Δy be the displacement of the spring in the vertical direction, then

$$mg = k\Delta y$$

Solving for Δy, we obtain

$$\Delta y = \frac{mg}{k} \tag{2}$$

SOLUTION According to equation (1') above, the spring constant *k* is given by

$$k = \frac{mv^2}{\Delta L(L_0 + \Delta L)}$$

Substituting this expression for *k* into equation (2) gives

$$\Delta y = \frac{mg\Delta L(L_0 + \Delta L)}{mv^2} = \frac{g\Delta L(L_0 + \Delta L)}{v^2}$$

or

$$\Delta y = \frac{(9.80\,\text{m/s}^2)(0.010\,\text{m})(0.200\,\text{m} + 0.010\,\text{m})}{(3.00\,\text{m/s})^2} = \boxed{2.29 \times 10^{-3}\ \text{m}}$$

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17. **mmh** A block of mass $m = 0.750$ kg is fastened to an unstrained horizontal spring whose spring constant is $k = 82.0$ N/m. The block is given a displacement of +0.120 m, where the + sign indicates that the displacement is along the $+x$ axis, and then released from rest. **(a)** What is the force (magnitude and direction) that the spring exerts on the block just before the block is released? **(b)** Find the angular frequency ω of the resulting oscillatory motion. **(c)** What is the maximum speed of the block? **(d)** Determine the magnitude of the maximum acceleration of the block.

REASONING The force F_x that the spring exerts on the block just before it is released is equal to $-kx$, according to Equation 10.2. Here k is the spring constant and x is the displacement of the spring from its equilibrium position. Once the block has been released, it oscillates back and forth with an angular frequency given by Equation 10.11 as $\omega = \sqrt{k/m}$, where m is the mass of the block. The maximum speed that the block attains during the oscillatory motion is $v_{max} = A\omega$ (Equation 10.8). The magnitude of the maximum acceleration that the block attains is $a_{max} = A\omega^2$ (Equation 10.10).

SOLUTION

a. The force F_x exerted on the block by the spring is

$$F_x = -kx = -(82.0\ \text{N/m})(0.120\ \text{m}) = \boxed{-9.84\ \text{N}} \qquad (10.2)$$

b. The angular frequency ω of the resulting oscillatory motion is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{82.0\ \text{N/m}}{0.750\ \text{kg}}} = \boxed{10.5\ \text{rad/s}} \qquad (10.11)$$

c. The maximum speed v_{max} is the product of the amplitude and the angular frequency:

$$v_{max} = A\omega = (0.120\ \text{m})(10.5\ \text{rad/s}) = \boxed{1.26\ \text{m/s}} \qquad (10.8)$$

d. The magnitude a_{max} of the maximum acceleration is

$$a_{max} = A\omega^2 = (0.120 \text{ m})(10.5 \text{ rad/s})^2 = \boxed{13.2 \text{ m/s}^2} \tag{10.10}$$

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27. A spring is hung from the ceiling. A 0.450-kg block is then attached to the free end of the spring. When released from rest, the block drops 0.150 m before momentarily coming to rest, after which it moves back upward. **(a)** What is the spring constant of the spring? **(b)** Find the angular frequency of the block's vibrations.

REASONING As the block falls, only two forces act on it: its weight and the elastic force of the spring. Both of these forces are conservative forces, so the falling block obeys the principle of conservation of mechanical energy. We will use this conservation principle to determine the spring constant of the spring. Once the spring constant is known, Equation 10.11, $\omega = \sqrt{k/m}$, may be used to find the angular frequency of the block's vibrations.

SOLUTION

a. The conservation of mechanical energy states that the final total mechanical energy E_f is equal to the initial total mechanical energy E_0, or $E_f = E_0$ (Equation 6.9a). The expression for the total mechanical energy of an object oscillating on a spring is given by Equation 10.14. Thus, the conservation of total mechanical energy can be written as

$$\underbrace{\tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\omega_f^2 + mgh_f + \tfrac{1}{2}ky_f^2}_{E_f} = \underbrace{\tfrac{1}{2}mv_0^2 + \tfrac{1}{2}I\omega_0^2 + mgh_0 + \tfrac{1}{2}ky_0^2}_{E_0}$$

Before going any further, let's simplify this equation by noting which variables are zero. Since the block starts and ends at rest, $v_f = v_0 = 0$ m/s. The block does not rotate, so its angular speed is zero, $\omega_f = \omega_0 = 0$ rad/s. Initially, the spring is unstretched, so that $y_0 = 0$ m. Setting these terms equal to zero in the equation above gives

$$mgh_f + \tfrac{1}{2}ky_f^2 = mgh_0$$

Solving this equation for the spring constant k, we have that

$$k = \frac{mg\left(h_0 - h_\text{f}\right)}{\frac{1}{2}y_\text{f}^2} = \frac{(0.450\text{ kg})\left(9.80\text{ m/s}^2\right)(0.150\text{ m})}{\frac{1}{2}(0.150\text{ m})^2} = \boxed{58.8\text{ N/m}}$$

b. The angular frequency ω of the block's vibrations depends on the spring constant k and the mass m of the block:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{58.8\text{ N/m}}{0.450\text{ kg}}} = \boxed{11.4\text{ rad/s}} \qquad (10.11)$$

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51. **ssm** A tow truck is pulling a car out of a ditch by means of a steel cable that is 9.1 m long and has a radius of 0.50 cm. When the car just begins to move, the tension in the cable is 890 N. How much has the cable stretched?

REASONING When the tow truck pulls the car out of the ditch, the cable stretches and a tension exists in it. This tension is the force that acts on the car. The amount ΔL that the cable stretches depends on the tension F, the length L_0 and cross-sectional area A of the cable, as well as Young's modulus Y for steel. All of these quantities are given in the statement of the problem, except for Young's modulus, which can be found by consulting Table 10.1.

SOLUTION Solving Equation 10.17, $F = Y\left(\frac{\Delta L}{L_0}\right)A$, for the change in length, we have

$$\Delta L = \frac{F L_0}{AY} = \frac{(890\text{ N})(9.1\text{ m})}{\pi\left(0.50\times10^{-2}\text{ m}\right)^2\left(2.0\times10^{11}\text{ N/m}^2\right)} = \boxed{5.2\times10^{-4}\text{ m}}$$
