Week 2 Recitation:

Chapter 2: Problems 5, 19, 25, 29, 33, 39, 49, 58.

5. GO The data in the following table describe the initial and final positions of a moving car. The elapsed time for each of the three pairs of positions listed in the table is 0.50 s. Review the concept of average velocity in Section 2.2 and then determine the average velocity (magnitude and direction) for each of the three pairs. Note that the algebraic sign of your answers will convey the direction.

	Initial position x_0	**Final position x**
(a)	+2.0 m	+6.0 m
(b)	+6.0 m	+2.0 m
(c)	–3.0 m	+7.0 m

REASONING According to Equation 2.2 $\left(\bar{v} = \dfrac{x - x_0}{t - t_0}\right)$, the average velocity $(\bar{v})$ is equal to the displacement $(x - x_0)$ divided by the elapsed time $(t - t_0)$, and the direction of the average velocity is the same as that of the displacement. The displacement is equal to the difference between the final and initial positions.

SOLUTION: Equation 2.2 gives the average velocity as

$$\bar{v} = \frac{x - x_0}{t - t_0}$$

Therefore, the average velocities for the three cases are:

(a) Average velocity = (6.0 m – 2.0 m)/(0.50 s) = $\boxed{+8.0\ \text{m/s}}$

(b) Average velocity = (2.0 m – 6.0 m)/(0.50 s) = $\boxed{-8.0\ \text{m/s}}$

(c) Average velocity = [7.0 m – (–3.0 m)]/(0.50 s) = $\boxed{+2.0 \times 10^{1}\ \text{m/s}}$

The algebraic sign of the answer conveys the direction in each case.

19. GO The initial velocity and acceleration of four moving objects at a given instant in time are given in the following table. Determine the final *speed* of each of the objects, assuming that the time elapsed since $t = 0$ s is 2.0 s.

	Initial velocity v_0	Acceleration a
(a)	+12 m/s	+3.0 m/s^2
(b)	+12 m/s	–3.0 m/s^2
(c)	–12 m/s	+3.0 m/s^2
(d)	–12 m/s	–3.0 m/s^2

REASONING When the velocity and acceleration vectors are in the same direction, the speed of the object increases in time. When the velocity and acceleration vectors are in opposite directions, the speed of the object decreases in time. (a) The initial velocity and acceleration are in the same direction, so the speed is increasing. (b) The initial velocity and acceleration are in opposite directions, so the speed is decreasing. (c) The initial velocity and acceleration are in opposite directions, so the speed is decreasing. (d) The initial velocity and acceleration are in the same direction, so the speed is increasing.

SOLUTION: The final velocity v is related to the initial velocity v_0, the acceleration a, and the elapsed time t through Equation 2.4 ($v = v_0 + a\,t$). The final velocities and speeds for the four moving objects are:

a. $v = 12\text{ m/s} + (3.0\text{ m/s}^2)(2.0\text{ s}) = 18\text{ m/s}$. The final speed is $\boxed{18\text{ m/s}}$.

b. $v = 12\text{ m/s} + (-3.0\text{ m/s}^2)(2.0\text{ s}) = 6.0\text{ m/s}$. The final speed is $\boxed{6.0\text{ m/s}}$.

c. $v = -12\text{ m/s} + (3.0\text{ m/s}^2)(2.0\text{ s}) = -6.0\text{ m/s}$. The final speed is $\boxed{6.0\text{ m/s}}$.

d. $v = -12\text{ m/s} + (-3.0\text{ m/s}^2)(2.0\text{ s}) = -18\text{ m/s}$. The final speed is $\boxed{18\text{ m/s}}$.

25. ssm A jogger accelerates from rest to 3.0 m/s in 2.0 s. A car accelerates from 38.0 to 41.0 m/s also in 2.0 s. **(a)** Find the acceleration (magnitude only) of the jogger. **(b)** Determine the acceleration (magnitude only) of the car. **(c)** Does the car travel farther than the jogger during the 2.0 s? If so, how much farther?

SSM ***REASONING AND SOLUTION***

a. The magnitude of the acceleration can be found from Equation 2.4 ($v = v_0 + at$) as

$$a = \frac{v - v_0}{t} = \frac{3.0 \text{ m/s} - 0 \text{ m/s}}{2.0 \text{ s}} = \boxed{1.5 \text{ m/s}^2}$$

b. Similarly the magnitude of the acceleration of the car is

$$a = \frac{v - v_0}{t} = \frac{41.0 \text{ m/s} - 38.0 \text{ m/s}}{2.0 \text{ s}} = \boxed{1.5 \text{ m/s}^2}$$

c. Assuming that the acceleration is constant, the displacement covered by the car can be found from Equation 2.9 ($v^2 = v_0^2 + 2ax$):

$$x = \frac{v^2 - v_0^2}{2a} = \frac{(41.0 \text{ m/s})^2 - (38.0 \text{ m/s})^2}{2(1.5 \text{ m/s}^2)} = 79 \text{ m}$$

Similarly, the displacement traveled by the jogger is

$$x = \frac{v^2 - v_0^2}{2a} = \frac{(3.0 \text{ m/s})^2 - (0 \text{ m/s})^2}{2(1.5 \text{ m/s}^2)} = 3.0 \text{ m}$$

Therefore, the car travels 79 m – 3.0 m = $\boxed{76 \text{ m}}$ further than the jogger.

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29. ssm A jetliner, traveling northward, is landing with a speed of 69 m/s. Once the jet touches down, it has 750 m of runway in which to reduce its speed to 6.1 m/s. Compute the average acceleration (magnitude and direction) of the plane during landing.

SSM ***REASONING AND SOLUTION*** The average acceleration of the plane can be found by solving Equation 2.9 $\left(v^2 = v_0^2 + 2ax\right)$ for a. Taking the direction of motion as positive, we have

$$a = \frac{v^2 - v_0^2}{2x} = \frac{(+6.1 \text{ m/s})^2 - (+69 \text{ m/s})^2}{2(+750 \text{ m})} = \boxed{-3.1 \text{ m/s}^2}$$

The minus sign indicates that the direction of the acceleration is opposite to the direction of motion, and the plane is slowing down.

***33.** **mmh** A car is traveling at 20.0 m/s, and the driver sees a traffic light turn red. After 0.530 s (the reaction time), the driver applies the brakes, and the car decelerates at 7.00 m/s^2. What is the stopping distance of the car, as measured from the point where the driver first sees the red light?

REASONING: The stopping distance is the sum of two parts. First, there is the distance the car travels at 20.0 m/s before the brakes are applied. According to Equation 2.2, this distance is the magnitude of the displacement and is the magnitude of the velocity times the time. Second, there is the distance the car travels while it decelerates as the brakes are applied. This distance is given by Equation 2.9, since the initial velocity, the acceleration, and the final velocity (0 m/s when the car comes to a stop) are given.

SOLUTION: With the assumption that the initial position of the car is $x_0 = 0$ m, Equation 2.2 gives the first contribution to the stopping distance as

$$\Delta x_1 = x_1 = vt_1 = (20.0 \text{ m/s})(0.530 \text{ s})$$

Solving Equation 2.9 $\left(v^2 = v_0^2 + 2ax\right)$ for x shows that the second part of the stopping distance is

$$x_2 = \frac{v^2 - v_0^2}{2a} = \frac{(0 \text{ m/s})^2 - (20.0 \text{ m/s})^2}{2\left(-7.00 \text{ m/s}^2\right)}$$

Here, the acceleration is assigned a negative value, because we have assumed that the car is traveling in the positive direction, and it is decelerating. Since it is decelerating, its acceleration points opposite to its velocity. The stopping distance, then, is

$$x_{\text{Stopping}} = x_1 + x_2 = (20.0 \text{ m/s})(0.530 \text{ s}) + \frac{(0 \text{ m/s})^2 - (20.0 \text{ m/s})^2}{2\left(-7.00 \text{ m/s}^2\right)} = \boxed{39.2 \text{ m}}$$

***39.** Refer to Multiple-Concept Example 5 to review a method by which this problem can be solved. You are driving your car, and the traffic light ahead turns red. You apply the brakes for 3.00 s, and the velocity of the car decreases to +4.50 m/s. The car's deceleration has a magnitude of 2.70 m/s^2 during this time. What is the car's displacement?

REASONING: Because the car is traveling in the $+x$ direction and decelerating, its acceleration is negative: $a = -2.70\ \text{m/s}^2$. The final velocity for the interval is given ($v = +4.50$ m/s), as well as the elapsed time ($t = 3.00$ s). Both the car's displacement x and its initial velocity v_0 at the instant braking begins are unknown.

Compare the list of known kinematic quantities (v, a, t) to the equations of kinematics for constant acceleration: $v = v_0 + at$ (Equation 2.4), $x = \frac{1}{2}\left(v_0 + v\right)t$ (Equation 2.7), $x = v_0 t + \frac{1}{2}at^2$ (Equation 2.8), and $v^2 = v_0^2 + 2ax$ (Equation 2.9). None of these four equations contains all three known quantities and the desired displacement x, and each of them contains the initial velocity v_0. Since the initial velocity is neither known nor requested, we can combine two kinematic equations to eliminate it, leaving an equation in which x is the only unknown quantity.

SOLUTION: For the first step, solve Equation 2.4 $\left(v = v_0 + at\right)$ for v_0:

$$v_0 = v - at \tag{1}$$

Substituting the expression for v_0 in Equation (1) into Equation 2.8 $\left(x = v_0 t + \frac{1}{2}at^2\right)$ yields an expression for the car's displacement solely in terms of the known quantities v, a, and t:

$$x = \left(v - at\right)t + \tfrac{1}{2}at^2 = vt - at^2 + \tfrac{1}{2}at^2$$

$$x = vt - \tfrac{1}{2}at^2 \tag{2}$$

Substitute the known values of v, a, and t into Equation (2):

$$x = \left(+4.50\ \text{m/s}\right)\left(3.00\ \text{s}\right) - \tfrac{1}{2}\left(-2.70\ \text{m/s}^2\right)\left(3.00\ \text{s}\right)^2 = \boxed{+25.7\ \text{m}}$$

Note: Equation (2) can also be obtained by combining Equation (1) with Equation 2.7 $\left[x = \frac{1}{2}\left(v_0 + v\right)t\right]$, or, with more effort, by combining Equation (1) with Equation 2.9 $\left(v^2 = v_0^2 + 2ax\right)$.

49. ssm A hot-air balloon is rising upward with a constant speed of 2.50 m/s. When the balloon is 3.00 m above the ground, the balloonist accidentally drops a compass over the side of the balloon. How much time elapses before the compass hits the ground?

SSM ***REASONING*** The initial velocity of the compass is +2.50 m/s. The initial position of the compass is 3.00 m and its final position is 0 m when it strikes the ground. The displacement of the compass is the final position minus the initial position, or y = –3.00 m. As the compass falls to the ground, its acceleration is the acceleration due to gravity, a = –9.80 m/s^2. Equation 2.8 $\left(y = v_0 t + \frac{1}{2} a t^2\right)$ can be used to find how much time elapses before the compass hits the ground.

SOLUTION: Starting with Equation 2.8, we use the quadratic equation to find the elapsed time.

$$t = \frac{-v_0 \pm \sqrt{v_0^2 - 4\left(\frac{1}{2}a\right)(-y)}}{2\left(\frac{1}{2}a\right)} = \frac{-(2.50 \text{ m/s}) \pm \sqrt{(2.50 \text{ m/s})^2 - 4\left(-4.90 \text{ m/s}^2\right)\left[-(-3.00 \text{ m})\right]}}{2\left(-4.90 \text{ m/s}^2\right)}$$

There are two solutions to this quadratic equation, t_1 = $\boxed{1.08 \text{ s}}$ and t_2 = –0.568 s. The second solution, being a negative time, is discarded.

***58.** Two stones are thrown simultaneously, one straight upward from the base of a cliff and the other straight downward from the top of the cliff. The height of the cliff is 6.00 m. The stones are thrown with the same speed of 9.00 m/s. Find the location (above the base of the cliff) of the point where the stones cross paths.

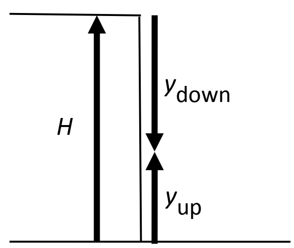
REASONING: The stone that is thrown upward loses speed on the way up. The stone that is thrown downward gains speed on the way down. The stones cross paths below the point that corresponds to half the height of the cliff. To see why, consider where they would cross paths if they each maintained their initial speed as they moved. Then, they would cross paths exactly at the halfway point. However, the stone traveling upward begins immediately to lose speed, while the stone traveling downward immediately gains speed. Thus, the upward moving stone travels more slowly than the downward moving stone. Consequently, the stone thrown downward has traveled farther when it reaches the crossing point than the stone thrown upward.

The initial velocity v_0 is known for both stones, as is the acceleration a due to gravity. In addition, we know that at the crossing point the stones are at the same place at the same time t. Furthermore, the position of each stone is specified by its displacement y from its starting point. The equation of kinematics that relates the variables v_0, a, t and y is Equation 2.8 $\left(y = v_0 t + \frac{1}{2}at^2\right)$, and we will use it in our solution. In using this equation, we will assume upward to be the positive direction.

SOLUTION: Applying Equation 2.8 to each stone, we have

$$\underbrace{y_{\text{up}} = v_0^{\text{up}}t + \tfrac{1}{2}at^2}_{\text{Upward moving stone}} \quad \text{and} \quad \underbrace{y_{\text{down}} = v_0^{\text{down}}t + \tfrac{1}{2}at^2}_{\text{Downward moving stone}}$$

In these expressions t is the time it takes for either stone to reach the crossing point, and a is the acceleration due to gravity. Note that y_{up} is the displacement of the upward moving stone above the base of the cliff, y_{down} is the displacement of the downward moving stone below the top of the cliff, and H is the displacement of the cliff-top above the base of the cliff, as the drawing shows. The distances above and below the crossing point must add to equal the height of the cliff, so we have



$$y_{\text{up}} - y_{\text{down}} = H$$

where the minus sign appears because the displacement y_{down} points in the negative direction. Substituting the two expressions for y_{up} and y_{down} into this equation gives

$$v_0^{\text{up}}t + \tfrac{1}{2}at^2 - \left(v_0^{\text{down}}t + \tfrac{1}{2}at^2\right) = H$$

This equation can be solved for t to show that the travel time to the crossing point is

$$t = \frac{H}{v_0^{\text{up}} - v_0^{\text{down}}}$$

Substituting this result into the expression from Equation 2.8 for y_{up} gives

$$y_{\text{up}} = v_0^{\text{up}} t + \tfrac{1}{2} a t^2 = v_0^{\text{up}} \left(\frac{H}{v_0^{\text{up}} - v_0^{\text{down}}} \right) + \tfrac{1}{2} a \left(\frac{H}{v_0^{\text{up}} - v_0^{\text{down}}} \right)^2$$

$$= (9.00\text{ m/s}) \left[\frac{6.00\text{ m}}{9.00\text{ m/s} - (-9.00\text{ m/s})} \right] + \tfrac{1}{2} \left(-9.80\text{ m/s}^2 \right) \left[\frac{6.00\text{ m}}{9.00\text{ m/s} - (-9.00\text{ m/s})} \right]^2$$

$$= 2.46\text{ m}$$

Thus, the crossing is located a distance of $\boxed{2.46\text{ m}}$ above the base of the cliff, which is below the halfway point of 3.00 m, as expected.