5. Suppose in Figure 6.2 that $+1.10 \times 10^3$ J of work is done by the force $\mathbf{F}$ (magnitude = 30.0 N) in moving the suitcase a distance of 50.0 m. At what angle $\theta$ is the force oriented with respect to the ground?

**SSM REASONING AND SOLUTION** Solving Equation 6.1 for the angle $\theta$, we obtain

$$
\theta = \cos^{-1}\left(\frac{W}{F s}\right) = \cos^{-1}\left[\frac{1.10 \times 10^3}{(30.0 \text{ N})(50.0 \text{ m})}\right] = 42.8^\circ
$$

6. A person pushes a 16.0-kg shopping cart at a constant velocity for a distance of 22.0 m. She pushes in a direction 29.0° below the horizontal. A 48.0-N frictional force opposes the motion of the cart. (a) What is the magnitude of the force that the shopper exerts? Determine the work done by (b) the pushing force, (c) the frictional force, and (d) the gravitational force.

**REASONING** The drawing shows three of the forces that act on the cart: $\mathbf{F}$ is the pushing force that the shopper exerts, $\mathbf{f}$ is the frictional force that opposes the motion of the cart, and $\mathbf{mg}$ is its weight. The displacement $\mathbf{s}$ of the cart is also shown. Since the cart moves at a constant velocity along the $+x$ direction, it is in equilibrium. The net force acting on it in this direction is zero, $\Sigma F_x = 0$. This relation can be used to find the magnitude of the pushing force. The work done by a constant force is given by Equation 6.1 as $W = (F \cos \theta)s$, where $F$ is the magnitude of the force, $s$ is the magnitude of the displacement, and $\theta$ is the angle between the force and the displacement.

**SOLUTION**

a. The $x$-component of the net force is zero, $\Sigma F_x = 0$, so that

$$
\frac{F \cos 29.0^\circ - f}{\Sigma F_x} = 0
$$

The magnitude of the force that the shopper exerts is

$$
F = \frac{f}{\cos 29.0^\circ} = \frac{48.0 \text{ N}}{\cos 29.0^\circ} = 54.9 \text{ N}.
$$

b. The work done by the pushing force $\mathbf{F}$ is

$$
W = (F \cos \theta)s = (54.9 \text{ N})(\cos 29.0^\circ)(22.0 \text{ m}) = 1060 \text{ J}
$$
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c. The angle between the frictional force and the displacement is 180°, so the work done by the frictional force $f$ is

$$W = (f \cos \theta) s = (48.0 \, \text{N})(\cos 180.0^\circ)(22.0 \, \text{m}) = -1060 \, \text{J}$$

d. The angle between the weight of the cart and the displacement is 90°, so the work done by the weight $mg$ is

$$W = (mg \cos \theta) s = (16.0 \, \text{kg})(9.80 \, \text{m/s}^2)(\cos 90^\circ)(22.0 \, \text{m}) = 0 \, \text{J}$$

**28.** Multiple-Concept Example 5 reviews many of the concepts that play roles in this problem. An extreme skier, starting from rest, coasts down a mountain slope that makes an angle of 25.0° with the horizontal. The coefficient of kinetic friction between her skis and the snow is 0.200. She coasts down a distance of 10.4 m before coming to the edge of a cliff. Without slowing down, she skis off the cliff and lands downhill at a point whose vertical distance is 3.50 m below the edge. How fast is she going just before she lands?

**REASONING** It is useful to divide this problem into two parts. The first part involves the skier moving on the snow. We can use the work-energy theorem to find her speed when she comes to the edge of the cliff. In the second part she leaves the snow and falls freely toward the ground. We can again employ the work-energy theorem to find her speed just before she lands.

**SOLUTION** The drawing at the right shows the three forces that act on the skier as she glides on the snow. The forces are: her weight $mg$, the normal force $F_N$, and the kinetic frictional force $f_k$. Her displacement is labeled as $s$. The work-energy theorem, Equation 6.3, is

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$$

where $W$ is the work done by the net external force that acts on the skier. The work done by each force is given by Equation 6.1, $W = (F \cos \theta)s$, so the work-energy theorem becomes

$$W = (mg \cos 65.0^\circ)s + (f_k \cos 180^\circ)s + (F_N \cos 90^\circ)s = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$$

Since $\cos 90^\circ = 0$, the third term on the left side can be eliminated. The magnitude $f_k$ of the kinetic frictional force is given by Equation 4.8 as $f_k = \mu_k F_N$. The magnitude $F_N$ of the normal force can be determined by noting that the skier does not leave the surface of the slope, so $a_y = 0 \, \text{m/s}^2$. Thus, we have that $\Sigma F_y = 0$, so
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\[ F_N - mg \cos 25.0^\circ = 0 \quad \text{or} \quad F_N = mg \cos 25.0^\circ \]

The magnitude of the kinetic frictional force becomes \( f_k = \mu_k F_N = \mu_k mg \cos 25.0^\circ \). Substituting this result into the work-energy theorem, we find that

\[
\frac{(mg \cos 65.0^\circ)s + \left(\mu_k mg \cos 25.0^\circ\right)(\cos 180^\circ)s}{W} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2
\]

Algebraically eliminating the mass \( m \) of the skier from every term, setting \( \cos 180^\circ = -1 \) and \( v_0 = 0 \) m/s, and solving for the final speed \( v_f \) gives

\[
v_f = \sqrt{2gs\left(\cos 65.0^\circ - \mu_k \cos 25.0^\circ\right)}
\]

\[
= \sqrt{2\left(9.80 \text{ m/s}^2\right)(10.4 \text{ m})\left[\cos 65.0^\circ - (0.200)\cos 25.0^\circ\right]} = 7.01 \text{ m/s}
\]

The drawing at the right shows her displacement \( s \) during free fall. Note that the displacement is a vector that starts where she leaves the slope and ends where she touches the ground. The only force acting on her during the free fall is her weight \( mg \). The work-energy theorem, Equation 6.3, is

\[ W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 \]

The work \( W \) is that done by her weight, so the work-energy theorem becomes

\[
\frac{(mg \cos \theta)s}{W} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2
\]

In this expression \( \theta \) is the angle between her weight (which points vertically downward) and her displacement. Note from the drawing that \( s \cos \theta = 3.50 \text{ m} \). Algebraically eliminating the mass \( m \) of the skier from every term in the equation above and solving for the final speed \( v_f \) gives

\[
v_f = \sqrt{v_0^2 + 2g(s \cos \theta)}
\]

\[
= \sqrt{(7.01 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)(3.50 \text{ m})} = 10.9 \text{ m/s}
\]

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38. The skateboarder in the drawing starts down the left side of the ramp with an initial speed of 5.4 m/s. Neglect nonconservative forces, such as friction and air resistance, and find the height $h$ of the highest point reached by the skateboarder on the right side of the ramp.

**REASONING** The distance $h$ in the drawing in the text is the difference between the skateboarder’s final and initial heights (measured, for example, with respect to the ground), or $h = h_f - h_0$. The difference in the heights can be determined by using the conservation of mechanical energy. This conservation law is applicable because non-conservative forces are negligible, so the work done by them is zero ($W_{nc} = 0 \text{ J}$). Thus, the skateboarder’s final total mechanical energy $E_f$ is equal to his initial total mechanical energy $E_0$:

$$\frac{1}{2} m v_f^2 + mgh_f = \frac{1}{2} m v_0^2 + mgh_0$$

(6.9b)

Solving Equation 6.9b for $h_f - h_0$, we find that

$$\frac{h_f - h_0}{h} = \frac{1}{2} \frac{v_0^2 - v_f^2}{g}$$

**SOLUTION** Using the fact that $v_0 = 5.4 \text{ m/s}$ and $v_f = 0 \text{ m/s}$ (since the skateboarder comes to a momentary rest), the distance $h$ is

$$h = \frac{1}{2} \frac{v_0^2 - v_f^2}{g} = \frac{1}{2} \frac{(5.4 \text{ m/s})^2 - (0 \text{ m/s})^2}{9.80 \text{ m/s}^2} = 1.5 \text{ m}$$
*43. [ssm] The drawing shows a skateboarder moving at 5.4 m/s along a horizontal section of a track that is slanted upward by 48° above the horizontal at its end, which is 0.40 m above the ground. When she leaves the track, she follows the characteristic path of projectile motion. Ignoring friction and air resistance, find the maximum height \( H \) to which she rises above the end of the track.

**SSM REASONING** To find the maximum height \( H \) above the end of the track we will analyze the projectile motion of the skateboarder after she leaves the track. For this analysis we will use the principle of conservation of mechanical energy, which applies because friction and air resistance are being ignored. In applying this principle to the projectile motion, however, we will need to know the speed of the skateboarder when she leaves the track. Therefore, we will begin by determining this speed, also using the conservation principle in the process. Our approach, then, uses the conservation principle twice.

**SOLUTION** Applying the conservation of mechanical energy in the form of Equation 6.9b, we have

\[
\frac{1}{2}mv_f^2 + mgh_f = \frac{1}{2}mv_0^2 + mgh_0
\]

We designate the flat portion of the track as having a height \( h_0 = 0 \) m and note from the drawing that its end is at a height of \( h_f = 0.40 \) m above the ground. Solving for the final speed at the end of the track gives

\[
v_f = \sqrt{v_0^2 + 2g(h_0 - h_f)} = \sqrt{(5.4 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)[(0 \text{ m}) - (0.40 \text{ m})]} = 4.6 \text{ m/s}
\]

This speed now becomes the initial speed \( v_0 = 4.6 \) m/s for the next application of the conservation principle. At the maximum height of her trajectory she is traveling horizontally with a speed \( v_f \) that equals the horizontal component of her launch velocity. Thus, for the next application of the conservation principle \( v_f = (4.6 \text{ m/s}) \cos 48° \). Applying the conservation of mechanical energy again, we have
**49.** A skier starts from rest at the top of a hill. The skier coasts down the hill and up a second hill, as the drawing illustrates. The crest of the second hill is circular, with a radius of $r = 36$ m. Neglect friction and air resistance. What must be the height $h$ of the first hill so that the skier just loses contact with the snow at the crest of the second hill?

**REASONING** If air resistance is ignored, the only non-conservative force that acts on the skier is the normal force exerted on the skier by the snow. Since this force is always perpendicular to the direction of the displacement, the work done by the normal force is zero. We can conclude, therefore, that mechanical energy is conserved. Our solution will be based on this fact.

**SOLUTION** The conservation of mechanical energy (Equation 6.9b) specifies that

$$\frac{1}{2}mv^2 + mgh = \frac{1}{2}mv_f^2 + mgh_f$$

Since the skier starts from rest $v_0 = 0$ m/s. Let $h_f$ define the zero level for heights, then the final gravitational potential energy is zero. This gives

$$mgh_0 = \frac{1}{2}mv_f^2$$

(1)

At the crest of the second hill, the two forces that act on the skier are the normal force and the weight of the skier. The resultant of these two forces provides the necessary centripetal force to keep the skier moving along the circular arc of the hill. When the skier just loses contact with the snow, the normal force is zero and the weight of the skier must provide the necessary centripetal force.
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\[ mg = \frac{mv^2_f}{r} \quad \text{so that} \quad v^2_f = gr \quad (2) \]

Substituting this expression for \( v^2_f \) into Equation (1) gives

\[ mgh_0 = \frac{1}{2} mrv^2 \quad \text{or} \quad h_0 = \frac{r}{2} = \frac{36 \, \text{m}}{2} = 18 \, \text{m} \]

53. Starting from rest, a 93-kg firefighter slides down a fire pole. The average frictional force exerted on him by the pole has a magnitude of 810 N, and his speed at the bottom of the pole is 3.4 m/s. How far did he slide down the pole?

**REASONING** As the firefighter slides down the pole from a height \( h_0 \) to the ground \( (h_f = 0 \, \text{m}) \), his potential energy decreases to zero. At the same time, his kinetic energy increases as he speeds up from rest \( (v_0 = 0 \, \text{m/s}) \) to a final speed \( v_f \). However, the upward nonconservative force of kinetic friction \( f_k \), acting over a downward displacement \( h_0 \), does a negative amount of work on him: \[ W_{nc} = \left( f_k \cos 180^\circ \right)h_0 = -f_k h_0 \quad \text{(Equation 6.1)}. \] This work decreases his total mechanical energy \( E \). Applying the work-energy theorem (Equation 6.8), with \( h_f = 0 \, \text{m} \) and \( v_0 = 0 \, \text{m/s} \), we obtain

\[ W_{nc} = \left( \frac{1}{2} mv^2_f + mgh_f \right) - \left( \frac{1}{2} mv^2_0 + mgh_0 \right) \quad (6.8) \]

\[ -f_k h_0 = \left( \frac{1}{2} mv^2_f + 0 \, \text{J} \right) - \left( 0 \, \text{J} + mgh_0 \right) = \frac{1}{2} mv^2_f - mgh_0 \quad (1) \]

**SOLUTION** Solving Equation (1) for the height \( h_0 \) gives

\[ mgh_0 - f_k h_0 = \frac{1}{2} mv^2_f \quad \text{or} \quad h_0 \left( mg - f_k \right) = \frac{1}{2} mv^2_f \quad \text{or} \quad h_0 = \frac{mv^2_f}{2 \left( mg - f_k \right)} \]

The distance \( h_0 \) that the firefighter slides down the pole is, therefore,

\[ h_0 = \frac{(93 \, \text{kg})(3.4 \, \text{m/s})^2}{2 \left[ (93 \, \text{kg})(9.80 \, \text{m/s}^2) - 810 \, \text{N} \right]} = 5.3 \, \text{m} \]