Chapter 6

Work and Energy

Work?
6.1 Work Done by a Constant Force

\[ W = Fs \]

Units? Joule

Scalar or Vector? Scalar

\[ W = (F \cos \theta)s \]

\[ \cos 0^\circ = 1 \]
\[ \cos 90^\circ = 0 \]
\[ \cos 180^\circ = -1 \]
Example 1  Pulling a Suitcase-on-Wheels
Find the work done if the force is 45.0-N, the angle is 50° and the displacement is 75.0 m (As shown in Fig).

\[ W = F \cos(\theta) \cdot s \]

2169.4 J
6.1 Work Done by a Constant Force

\[ W = (F \cos 0)s = Fs \]

\[ W = (F \cos 180)s = -Fs \]
Example: Accelerating a Crate
The truck is accelerating at a rate of $+1.50 \text{ m/s}^2$. The mass of the crate is 120-kg and it does not slip. The magnitude of the displacement is 65 m. What is the total work done on the crate by all of the forces acting on it?

The angle between the displacement and the normal force is? $90^\circ$

$W$ done by $F_N = 0$

The angle between the displacement and the weight is? $90^\circ$

$W$ done by $Weight = 0$
The angle between the displacement and the friction force is $0^\circ$.

Work done by friction $W = f_s \cdot s$

Since Crate does not slip

\[ f_s = \sum F = ma \]

\[ f_s = 120 \text{ kg} \times 1.5 \frac{m}{s^2} = 180 \text{ N} \]

Work done by friction $W = f_s \cdot s$

\[ = 180N \times 65 \text{ m} = 11,700 \text{ J} \]
6.2 The Work-Energy Theorem and Kinetic Energy

Consider a constant net external force acting on an object. The object is displaced a distance $s$, in the same direction as the net force.

$$
\sum F = ma \quad s \quad W = \left( \sum F \right)s = (ma)s
$$

$$
W = m(as) = m \frac{1}{2} \left( v_f^2 - v_o^2 \right) = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_o^2
$$

$$
v_f^2 = v_o^2 + 2(ax) \quad (ax) = \frac{1}{2} \left( v_f^2 - v_o^2 \right)
$$

DEFINITION OF KINETIC ENERGY

The kinetic energy $KE$ of an object with mass $m$ and speed $v$ is given by

$$
KE = \frac{1}{2} mv^2
$$

$$
W = KE_f - KE_o
$$
The Work-Energy Theorem and Kinetic Energy

When a net external force does work on an object, the kinetic energy of the object changes by the amount of work done on it:

\[ W = KE_f - KE_o = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_o^2 \]

\[ \Sigma Fs = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_o^2 \]
Example: Deep Space
The mass of the space probe is 474-kg and its initial velocity is 275 m/s. If the 56.0-mN force acts on the probe through a displacement of $2.42 \times 10^9$ m, what is its final speed?
6.2 The Work-Energy Theorem and Kinetic Energy

\[ KE_0 = \frac{1}{2} m v_0^2 = 1.79 \times 10^7 \, J \]

\[ W = F \cdot s = 1.355 \times 10^8 \, J \]

\[ KE_f = 1.534 \times 10^8 \, J \]

\[ KE_f = \frac{1}{2} m v_f^2 \quad \Rightarrow \quad v_f = \sqrt{\frac{2KE_f}{m}} \]

\[ W = KE_f - KE_o \]

\[ KE_f = W + KE_o \]

\[ 805 \, \text{m/s} \]
Example: Downhill Skiing

In this case the net force along x-direction is

\[ \sum F = mg \sin 25^\circ - f_k \]
6.3 Gravitational Potential Energy

\[ W = (F \cos \theta)s \]

Work done by gravity

\[ W_{\text{gravity}} = mg(h_o - h_f) \]
6.3 Gravitational Potential Energy

DEFINITION OF GRAVITATIONAL POTENTIAL ENERGY

The gravitational potential energy $PE$ is the energy that an object of mass $m$ has by virtue of its position relative to the surface of the earth. That position is measured by the height $h$ of the object relative to an arbitrary zero level:

$$PE = mgh$$

While $KE$ is

$$KE = \frac{1}{2}mv^2$$

Work done by a force $F$ is

$$W = (F \cos \theta)s$$

Work-Energy Theorem

$$W = \Delta KE$$
The gravitational potential energy \( PE \) is the energy that an object of mass \( m \) has by virtue of its position relative to the surface of the earth. That position is measured by the height \( h \) of the object relative to an arbitrary zero level:

\[
PE = mgh
\]

**DEFINITION OF GRAVITATIONAL POTENTIAL ENERGY**

The gravitational potential energy \( PE \) is the energy that an object of mass \( m \) has by virtue of its position relative to the surface of the earth. That position is measured by the height \( h \) of the object relative to an arbitrary zero level:

\[
PE = mgh
\]
6.4 Conservative Versus Nonconservative Forces

DEFINITION OF A CONSERVATIVE FORCE

**Version 1**  A force is conservative when the work done by the force on an object is independent of the path between the object’s initial and final positions.

**Version 2**  A force is conservative when it does no work on an object moving around a closed path, starting and finishing at the same point.
Version 1  A force is conservative when the work it does on a moving object is independent of the path between the object’s initial and final positions.

\[ W_{\text{gravity}} = mg(h_o - h_f) \]
Version 2  A force is conservative when it does no work on an object moving around a closed path, starting and finishing at the same point.

\[ W_{\text{gravity}} = mg(h_o - h_f) \]

\[ h_o = h_f \]
## 6.4 Conservative Versus Nonconservative Forces

<table>
<thead>
<tr>
<th>Table 6.2</th>
<th>Some Conservative and Nonconservative Forces</th>
</tr>
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<tbody>
<tr>
<td><strong>Conservative Forces</strong></td>
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<td>Gravitational force (Ch. 4)</td>
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<td>Elastic spring force (Ch. 10)</td>
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<td>Electric force (Ch. 18, 19)</td>
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<td><strong>Nonconservative Forces</strong></td>
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<tr>
<td>Static and kinetic frictional forces</td>
<td></td>
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<tr>
<td>Air resistance</td>
<td></td>
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<tr>
<td>Tension</td>
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<tr>
<td>Normal force</td>
<td></td>
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<tr>
<td>Propulsion force of a rocket</td>
<td></td>
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</tbody>
</table>
6.4 Conservative Versus Non-conservative Forces

In normal situations both conservative and non-conservative forces act simultaneously on an object, so the work done by the net external force can be written as

\[ W = W_c + W_{nc} \]

\[ W = KE_f - KE_o = \Delta KE \]

\[ W_c = W_{\text{gravity}} = mgh_o - mgh_f = PE_o - PE_f = -\Delta PE \]
6.4 Conservative Versus Nonconservative Forces

\[ W = W_c + W_{nc} \]

\[ \Delta KE = -\Delta PE + W_{nc} \]

RESTATING THE WORK-ENERGY THEOREM

\[ W_{nc} = \Delta KE + \Delta PE \]

Change in Mechanical Energy
6.5 The Conservation of Mechanical Energy

\[ W_{nc} = \Delta KE + \Delta PE = (KE_f - KE_o) + (PE_f - PE_o) \]

\[ W_{nc} = (KE_f + PE_f) - (KE_o + PE_o) \]

\[ W_{nc} = E_f - E_o \]

**Principle of conservation of energy**

If the net work done on an object by non-conservative forces is zero, its mechanical energy does not change (remains conserved):

\[ E_f = E_o \]
6.5 The Conservation of Mechanical Energy

THE PRINCIPLE OF CONSERVATION OF MECHANICAL ENERGY

The total mechanical energy \( (E = KE + PE) \) of an object remains constant as the object moves, provided that the net work done by external non-conservative forces is zero.

\[
E_f = E_o \quad OR \quad \Delta E = E_f - E_o = 0
\]

\[
(mgh_f + \frac{1}{2}mv_f^2) = (mgh_o + \frac{1}{2}mv_o^2)
\]
### 6.5 The Conservation of Mechanical Energy

<table>
<thead>
<tr>
<th>KE</th>
<th>PE</th>
<th>$E = KE + PE$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 J</td>
<td>600 000 J</td>
<td>600 000 J</td>
</tr>
<tr>
<td>200 000 J</td>
<td>400 000 J</td>
<td>600 000 J</td>
</tr>
<tr>
<td>400 000 J</td>
<td>200 000 J</td>
<td>600 000 J</td>
</tr>
<tr>
<td>600 000 J</td>
<td>0 J</td>
<td>600 000 J</td>
</tr>
</tbody>
</table>

$\vec{v}_0 = 0 \text{ m/s}$
**6.7 Power**

**DEFINITION OF AVERAGE POWER**

Average power is the rate at which work is done, and it is obtained by dividing the work by the time required to perform the work.

\[ \bar{P} = \frac{\text{Work}}{\text{Time}} = \frac{W}{t} \]

Units?

joule/s = watt (W)

\[ \bar{P} = \frac{\text{Change in Energy}}{\text{Time}} = \frac{\Delta E}{t} \]

\[ \bar{P} = F \bar{v} \]

1 horsepower = 550 foot \cdot pounds/second = 745.7 watts
### Table 6.4  Human Metabolic Rates

<table>
<thead>
<tr>
<th>Activity</th>
<th>Rate (watts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Running (15 km/h)</td>
<td>1340 W</td>
</tr>
<tr>
<td>Skiing</td>
<td>1050 W</td>
</tr>
<tr>
<td>Biking</td>
<td>530 W</td>
</tr>
<tr>
<td>Walking (5 km/h)</td>
<td>280 W</td>
</tr>
<tr>
<td>Sleeping</td>
<td>77 W</td>
</tr>
</tbody>
</table>

*aFor a young 70-kg male.*
6.5 The Conservation of Mechanical Energy

Conceptual Example: The Favorite Swimming Hole

The person starts from rest, with the rope held in the horizontal position, swings downward, and then lets go of the rope. Three forces act on him: his weight, the tension in the rope, and the force of air resistance.

Can the principle of conservation of energy be used to calculate his final speed?

YES, if air resistance is ignored.
**Example:** Find the final speed of the person, if she starts from rest at a height of 4.0 m above the water surface and goes of the rope when she was 1.0 m above the water surface. (ignore air resistance)

\[
E_0 = PE_0 + KE_0
\]

\[
E_0 = PE_0 + KE_0
\]

\[
E_0 = mgh_0
\]

\[
E_f = PE_f + KE_f
\]

\[
E_f = mgh_f + \frac{1}{2}mv_f^2
\]

\[
E_0 = E_f
\]

\[
mgh_0 = mgh_f + \frac{1}{2}mv_f^2
\]

\[
mg(h_o - h_f) = \frac{1}{2}mv_f^2
\]

\[
v_f = \sqrt{2g(h_o - h_f)}
\]

\[
7.7 \text{ m/s}
\]
Gravitational Potential Energy

Example: A Gymnast on a Trampoline
The gymnast leaves the trampoline at an initial height of 1.20 m and reaches a maximum height of 4.80 m before falling back down. What was the initial speed of the gymnast?

\[ v_f = 0 \text{ m/s} \]

\[ h_f = \text{final height} = 4.80 \text{ m} \]

\[ h_0 = \text{initial height} = 1.20 \text{ m} \]

\[ v_0 = \text{initial speed} \]

\[ v_f = v_0 \cos \theta \]

\[ v_0 = \frac{v_f}{\cos \theta} = \frac{0}{\cos \theta} = 0 \text{ m/s} \]

\[ v_0 = \frac{\sqrt{2g(h_f - h_0)}}{\cos \theta} \]

\[ v_0 = \frac{\sqrt{2 \times 9.81 \text{ m/s}^2 \times (4.80 \text{ m} - 1.20 \text{ m})}}{\cos \theta} \]

\[ v_0 = \frac{\sqrt{2 \times 9.81 \times 3.60 \text{ m}^2}}{\cos \theta} \]

\[ v_0 = \frac{\sqrt{70.46 \text{ m}^2}}{\cos \theta} \]

\[ v_0 = \frac{8.4 \text{ m/s}}{\cos \theta} \]

\[ \cos \theta = \frac{1}{\sqrt{1 + \tan^2 \theta}} \]

\[ \cos \theta = \frac{1}{\sqrt{1 + (\frac{v_y}{v_0})^2}} \]

\[ \cos \theta = \frac{1}{\sqrt{1 + (\frac{9.81 \text{ m/s}^2}{v_0 \text{ m/s}})^2}} \]

\[ \cos \theta = \frac{1}{\sqrt{1 + (\frac{9.81}{8.4})^2}} \]

\[ \cos \theta = \frac{1}{\sqrt{1 + 1.21}} \]

\[ \cos \theta = \frac{1}{\sqrt{2.21}} \]

\[ \cos \theta = 0.486 \]

\[ v_0 = \frac{8.4 \text{ m/s}}{0.486} = 17.27 \text{ m/s} \]

\[ v_0 = 8.4 \text{ m/s} \]

\[ \boxed{8.4 \text{ m/s}} \]
Example: A Daredevil Motorcyclist
A motorcyclist is trying to leap across the canyon by driving horizontally off a cliff 38.0 m/s. Ignoring air resistance, find the speed with which the cycle strikes the ground on the other side.
6.5 The Conservation of Mechanical Energy

Loss in \( PE \) = Gain in \( KE \)

\[
\text{Loss in } PE = mg(h_o - h_f) \quad \quad \quad \quad \quad \quad \text{Gain in } KE = \frac{1}{2} m(v_f^2 - v_o^2)
\]

\[
\frac{1}{2} m (v_f^2 - v_o^2) = mg(h_o - h_f)
\]

\[
v_f = \sqrt{[v_o^2 + 2g\Delta h]} \quad \quad \quad \quad 46.15 \text{ m/s}
\]
- **Understanding your electricity bill**

  Companies bill you based upon Units you spend

  What is 1 Unit?

  \[ 1 \text{ kWh} = 1 \times 10^3 \frac{J}{s} \text{ h} \]

  \[ = 1 \times 10^3 \frac{J}{s} (3600 \text{ s}) \]

  Therefore

  \[ 1 \text{ kWh} = 3.6 \times 10^6 J \]

- **Your Car Engine**

  \[ 1 \text{ hp (horsepower)} = 745.7 \text{ W} \]

  1000 hp engine uses \[ 745.7 \times 1000 J = 7.457 \times 10^6 J \] of energy each second.
Summary

- $KE = \frac{1}{2} mv^2$
- $PE = mgh$
- $E = KE + PE$
- $W = \Delta E$
- $\bar{P} = \frac{W}{t} = F \bar{v}$

- Conservative and non-conservative forces
- Principle of Energy Conservation

Energy can not be created or destroyed but it can transform from one form to another form.

For Recitation Practice

**Chapter 6**: FOC 1, 2, 8, 11, 21 & 25.
Problems 5, 6, 38, 43, & 49.