

# *Chapter 7*

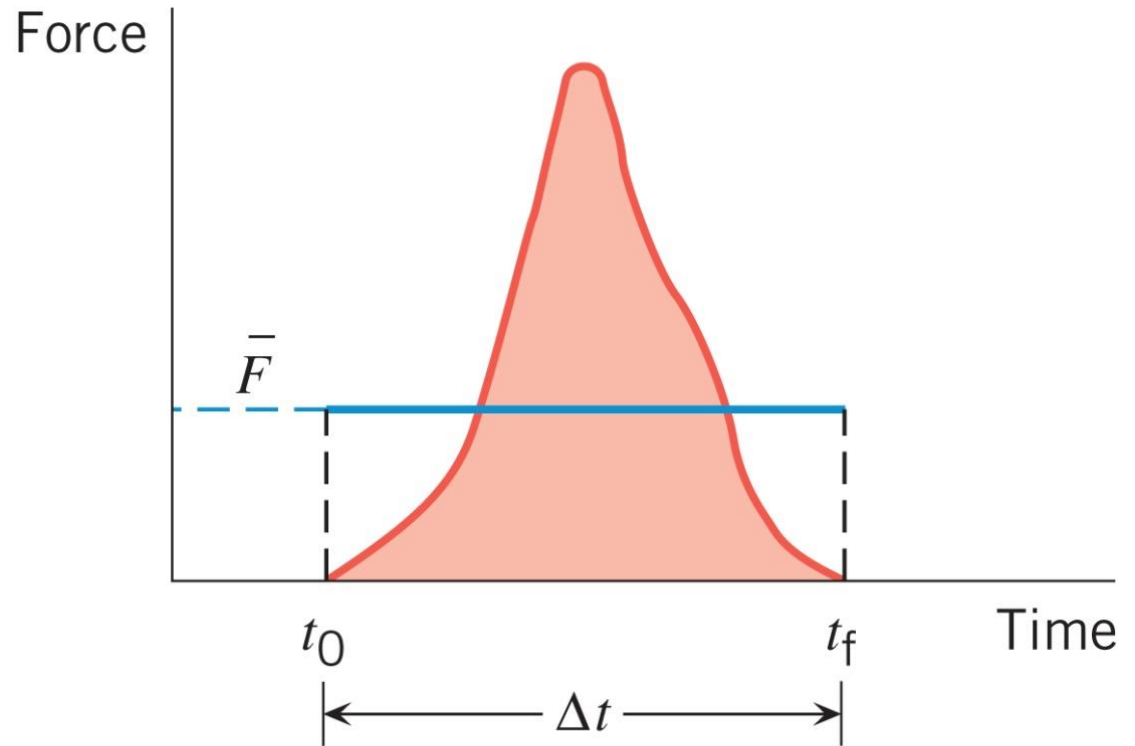
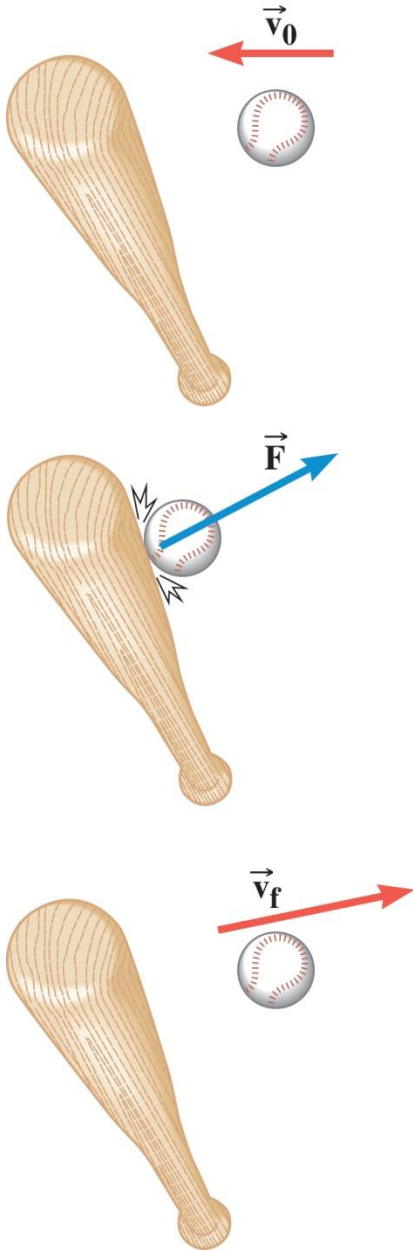
## *Impulse and Momentum*

So far we considered only constant force/s

**BUT**

There are many situations when the force  
on an object is not constant

# Force varies with time



### DEFINITION OF IMPULSE

The impulse of a force is the product of the average force and the time interval during which the force acts.

$$\vec{\mathbf{J}} = \vec{\mathbf{F}} \Delta t$$

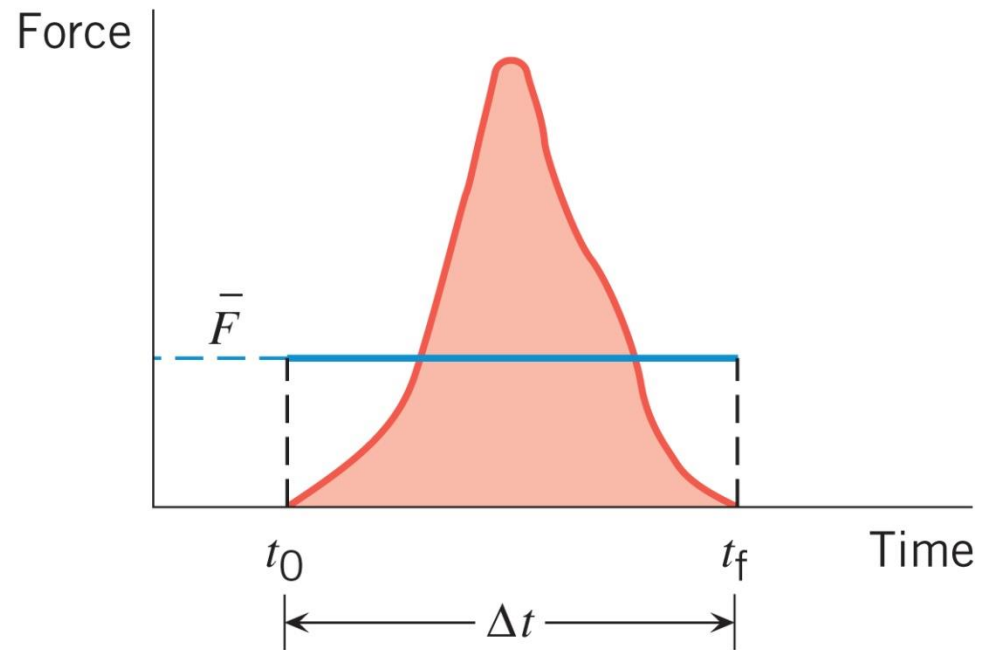
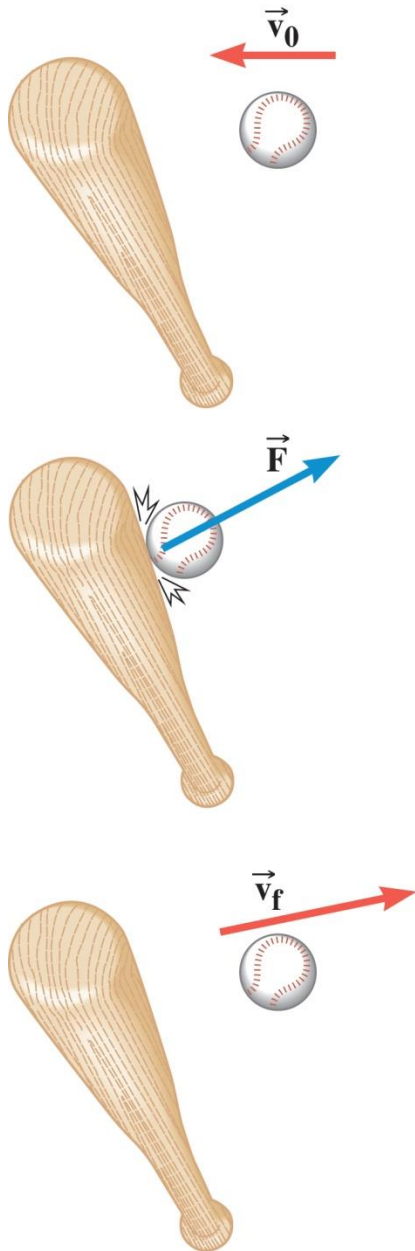
Impulse is a vector quantity and has the same direction as the average force.

Units?

Newton · seconds (N · s)

$$\text{kg} \frac{\text{m}}{\text{s}}$$

## 7.1 The Impulse-Momentum Theorem



$$\vec{\mathbf{J}} = \vec{\mathbf{F}} \Delta t$$

### DEFINITION OF LINEAR MOMENTUM

The linear momentum of an object is the product of the object's mass times its velocity

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$$

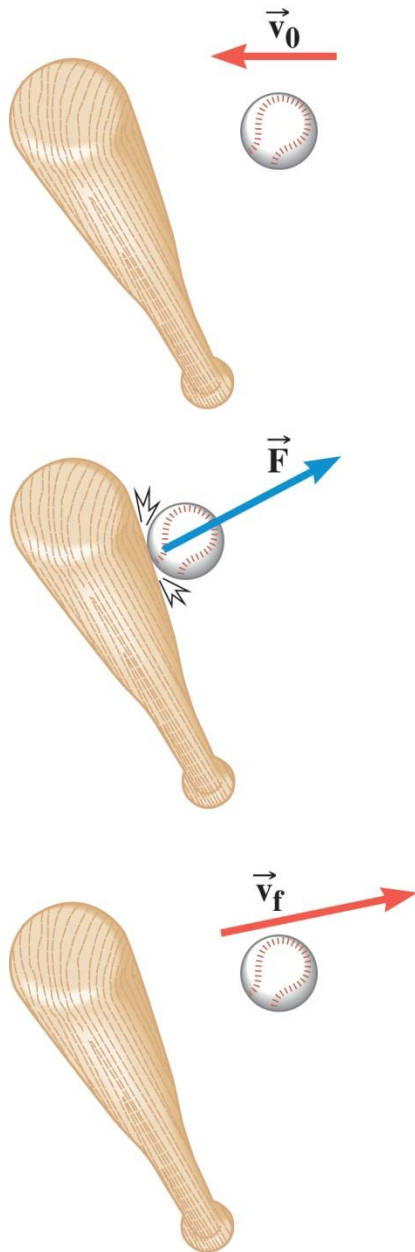
Momentum is a vector quantity and has the same direction as the velocity

Units?

kilogram · meter/second (kg · m/s)

N · s

## 7.1 The Impulse-Momentum Theorem



$$\vec{a} = \frac{\vec{v}_f - \vec{v}_0}{\Delta t}$$

$$\sum \vec{F} = m\vec{a}$$

$$\sum \vec{F} = m \left( \frac{\vec{v}_f - \vec{v}_0}{\Delta t} \right)$$

$$\left( \sum \vec{F} \right) \Delta t = m\vec{v}_f - m\vec{v}_0$$

$$\vec{J} = \vec{p}_f - \vec{p}_0$$

## IMPULSE-MOMENTUM THEOREM

When a net force acts on an object, the impulse of this force is equal to the change in the linear momentum of the object

$$\vec{\mathbf{J}} = \overline{\Delta \mathbf{p}}$$

Original form of Newton's Law of motion

$$\left( \sum \overline{\vec{\mathbf{F}}} \right) \Delta t$$

$$m\vec{\mathbf{v}}_f - m\vec{\mathbf{v}}_o$$

final momentum

initial momentum

$$\left( \sum \overline{\vec{\mathbf{F}}} \right) \Delta t = m\vec{\mathbf{v}}_f - m\vec{\mathbf{v}}_o$$

**Example: A Rain Storm**

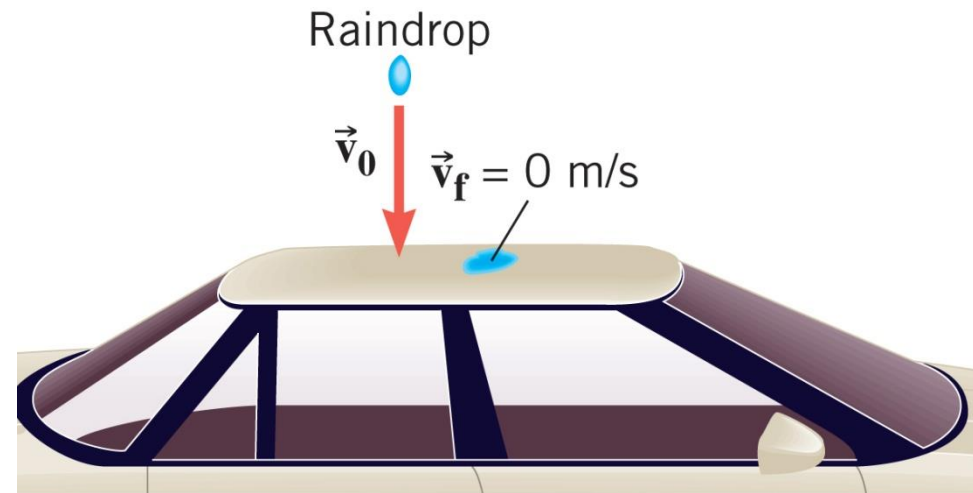
Rain comes down with a velocity of  $-15 \text{ m/s}$  and hits the roof of a car. The mass of rain per second that strikes the roof of the car is  $0.060 \text{ kg/s}$ . Assuming that rain comes to rest upon striking the car, find the average force exerted by the rain on the roof.

$$\left(\sum \vec{\mathbf{F}}\right)\Delta t = m\vec{\mathbf{v}}_f - m\vec{\mathbf{v}}_o$$

$$\vec{\mathbf{F}} = -\left(\frac{m}{\Delta t}\right)\vec{\mathbf{v}}_o$$

$$\vec{\mathbf{F}} = -(0.060 \text{ kg/s})(-15 \text{ m/s})$$

$$\boxed{+0.90 \text{ N}}$$



## 7.2 The Principle of Conservation of Linear Momentum

WORK-ENERGY THEOREM (Ch. 6)



CONSERVATION OF ENERGY

IMPULSE-MOMENTUM THEOREM

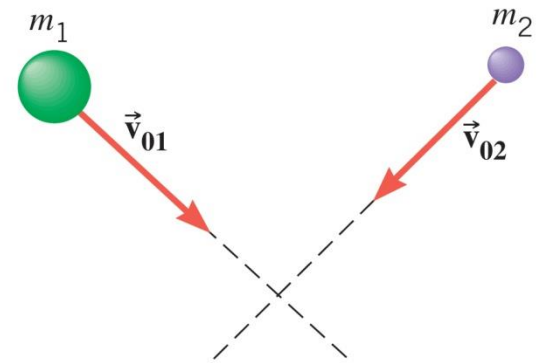


???

Apply the impulse-momentum theorem to the midair collision between two objects

**Internal forces** – Forces that objects within the system exert on each other.

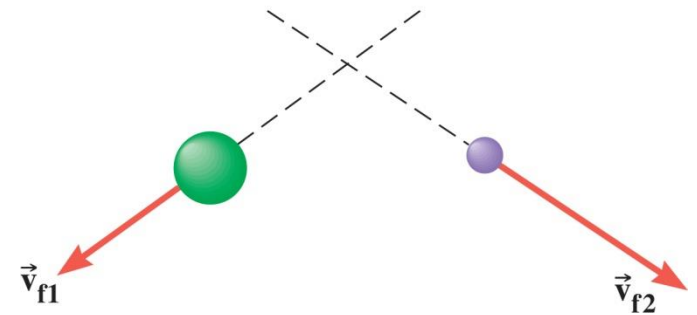
**External forces** – Forces exerted on objects by agents external to the system.



(a) Before collision



(b) During collision



(c) After collision

## 7.2 The Principle of Conservation of Linear Momentum

### Impulse-Momentum Theorem

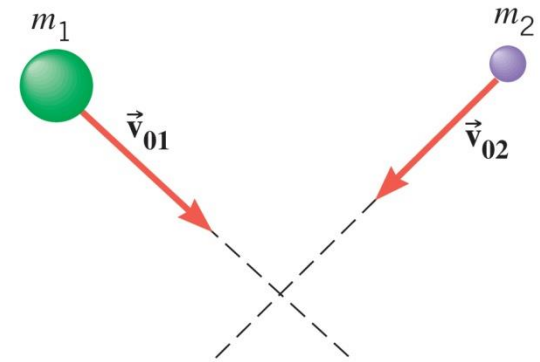
$$\left(\sum \vec{F}\right)\Delta t = m\vec{v}_f - m\vec{v}_o$$

### OBJECT 1

$$\left(\vec{W}_1 + \vec{F}_{12}\right)\Delta t = m_1\vec{v}_{f1} - m_1\vec{v}_{o1}$$

### OBJECT 2

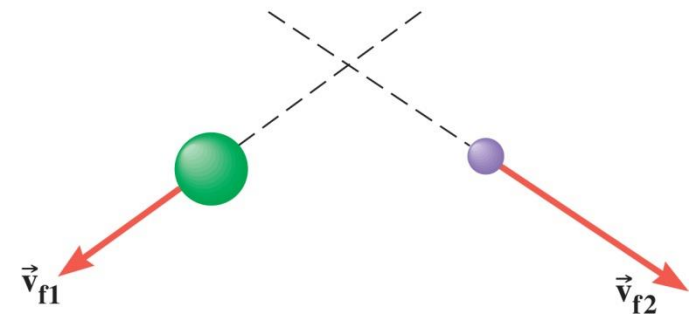
$$\left(\vec{W}_2 + \vec{F}_{21}\right)\Delta t = m_2\vec{v}_{f2} - m_2\vec{v}_{o2}$$



(a) Before collision



(b) During collision



(c) After collision

## 7.2 The Principle of Conservation of Linear Momentum

$$\left(\vec{\mathbf{W}}_1 + \vec{\mathbf{F}}_{12}\right)\Delta t = m_1 \vec{\mathbf{v}}_{f1} - m_1 \vec{\mathbf{v}}_{o1}$$

+

$$\left(\vec{\mathbf{W}}_2 + \vec{\mathbf{F}}_{21}\right)\Delta t = m_2 \vec{\mathbf{v}}_{f2} - m_2 \vec{\mathbf{v}}_{o2}$$



$$\left(\vec{\mathbf{W}}_1 + \vec{\mathbf{W}}_2 + \vec{\mathbf{F}}_{12} + \vec{\mathbf{F}}_{21}\right)\Delta t = \left(m_1 \vec{\mathbf{v}}_{f1} + m_2 \vec{\mathbf{v}}_{f2}\right) - \left(m_1 \vec{\mathbf{v}}_{o1} + m_2 \vec{\mathbf{v}}_{o2}\right)$$

But

$$\vec{\mathbf{F}}_{12} = -\vec{\mathbf{F}}_{21}$$

$$\vec{\mathbf{p}}_f$$

$$\vec{\mathbf{p}}_o$$

The internal forces cancel out.

$$\left(\vec{\mathbf{W}}_1 + \vec{\mathbf{W}}_2\right)\Delta t = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_o$$

$$\left(\text{sum of external forces}\right)\Delta t = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_o$$

## 7.2 The Principle of Conservation of Linear Momentum

$$(\text{sum of average external forces})\Delta t = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_o$$

If the sum of the external forces is zero, then

$$0 = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_o \quad \Rightarrow \quad \vec{\mathbf{p}}_f = \vec{\mathbf{p}}_o$$

### PRINCIPLE OF CONSERVATION OF LINEAR MENTUM

If the sum of external forces is zero, the total linear momentum of the system is constant (conserved)

The total linear momentum of an isolated system is constant (conserved).

An **isolated system** is one for which the sum of the external forces acting on the system is zero.

### Applying the Principle of Conservation of Linear Momentum

1. Decide which objects are included in the system.
2. Relative to the system, identify the internal and external forces.
3. Verify that the system is isolated.
4. Set the final momentum of the system equal to its initial momentum.

#### **Remember**

- Momentum is a vector

Decide positive and negative directions in the start.

**Question:** A ball of mass  $m$  hits a wall horizontally with speed  $v$  and bounces back with same speed. Which of the following is right to say about linear momentum and KE of the ball?

- a) Both momentum and KE of the ball are conserved.
- b) Both momentum and KE of the ball are not conserved.
- c) Only momentum of the ball is conserved.
- d) Only KE of the ball is conserved.

Isolated system?

**Example:** Starting from rest, two skaters push off against each other on ice where friction is negligible.

One is a 54-kg woman and other is 88-kg man. The woman moves away with a speed of +2.5 m/s. Find the recoil velocity of the man.

7.2 The Principle of Conservation of Linear Momentum

**Example:** Starting from rest, two skaters push off against each other on ice where friction is negligible.

One is a 54-kg woman and other is 88-kg man. The woman moves away with a speed of +2.5 m/s. Find the recoil velocity of the man.

System here is “both skaters”

Before the push

$$\vec{p}_0 = m_1 \vec{v}_{o1} + m_2 \vec{v}_{o2} \quad \boxed{\vec{p}_0 = 0}$$

After the push

$$\vec{p}_f = m_1 \vec{v}_{f1} + m_2 \vec{v}_{f2}$$

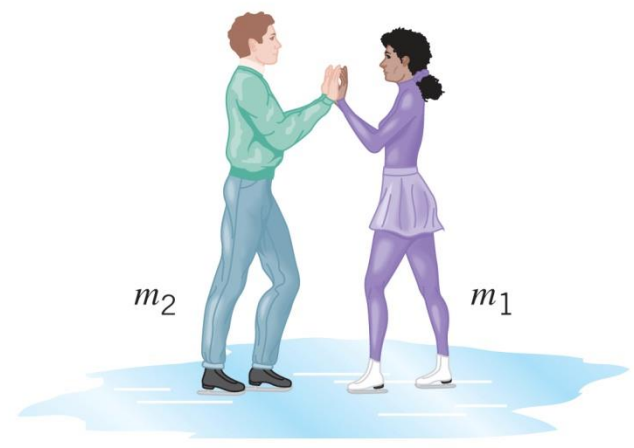
Conservation of Linear momentum

$$\boxed{\vec{p}_f = \vec{p}_0}$$

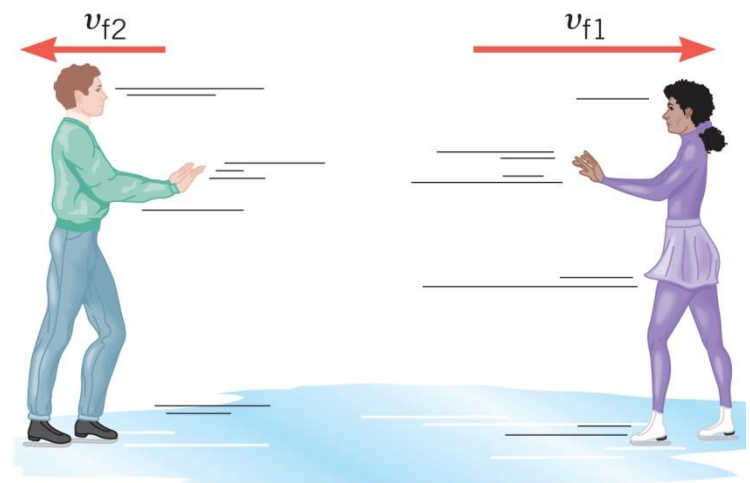
$$m_1 \vec{v}_{f1} + m_2 \vec{v}_{f2} = 0$$

$$v_{f2} = - \frac{m_1}{m_2} v_{f1}$$

$$\boxed{-1.53 \text{ m/s}}$$



(a) Before pushoff



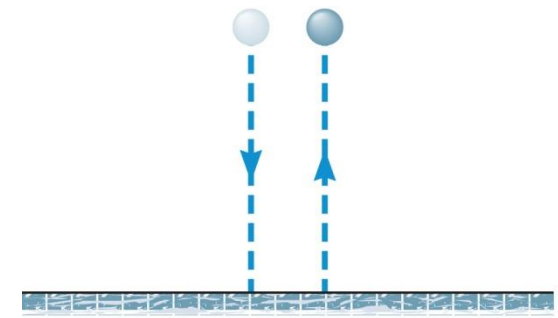
(b) After pushoff

**Collision:** The total linear momentum is conserved when two or more objects collide, provided they constitute an isolated system.

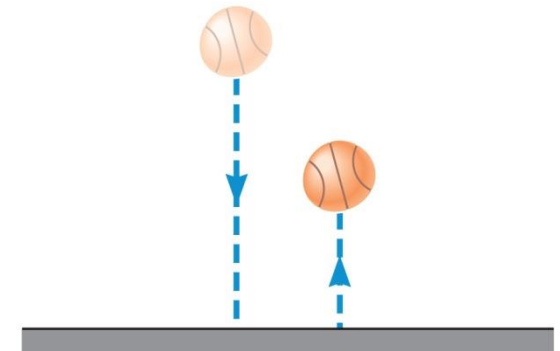
## Two Types of Collision

**Elastic collision** -- One in which the total kinetic energy of the system after the collision is equal to the total kinetic energy before the collision.

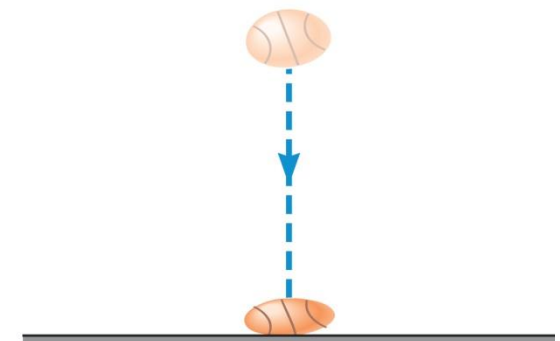
**Inelastic collision** -- One in which the total kinetic energy of the system after the collision is *not* equal to the total kinetic energy before the collision; if the objects stick together after colliding, the collision is said to be **completely inelastic**.



(a) Elastic collision



(b) Inelastic collision



(c) Completely inelastic collision

## 7.3 Collisions in One Dimension

### **Example: A Ballistic Pendulum**

The mass of the block of wood is 2.50-kg and the mass of the bullet is 0.0100-kg. The block swings to a maximum height of 0.650 m above the initial position. Find the initial speed of the bullet.

System? **Bullet and block**

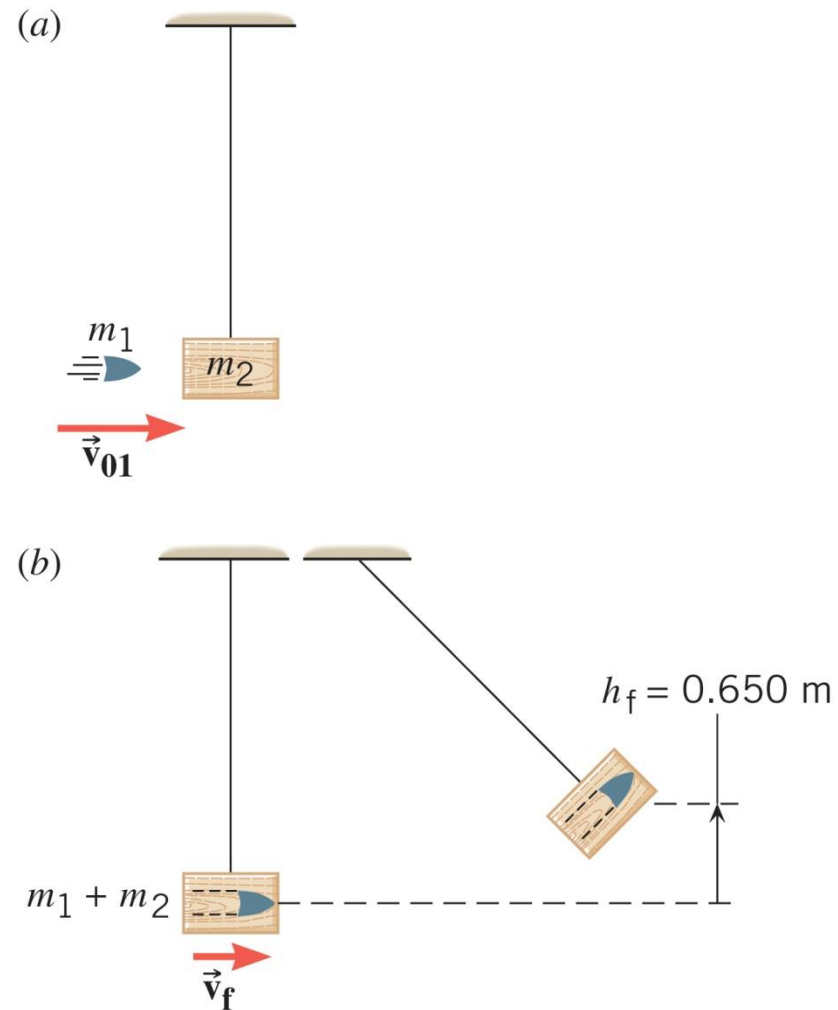
Linear Momentum will be conserved

Type of collision?

**Completely Inelastic**

KE will not be conserved

But Mechanical Energy will be conserved



**7.3 Collisions in One Dimension**

**Example: A Ballistic Pendulum**

The mass of the block of wood is 2.50-kg and the mass of the bullet is 0.0100-kg. The block swings to a maximum height of 0.650 m above the initial position.

Find the initial speed of the bullet.

**Before Collision**

$$p_0 = m_1 v_{o1} + m_2 v_{o2} \quad \boxed{p_0 = m_1 v_{o1}}$$

**After Collision**

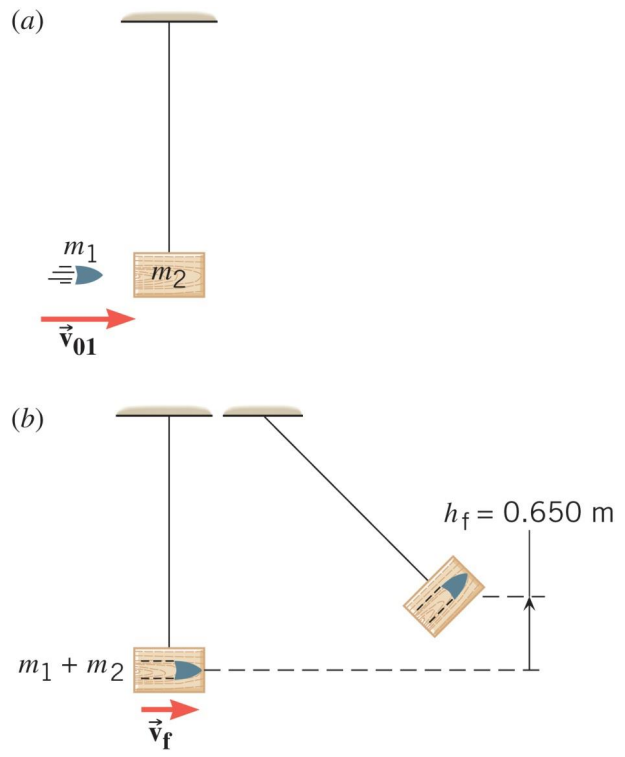
$$p_f = (m_1 + m_2) v_f$$

Linear Momentum will be conserved

$$p_f = p_0$$

$$(m_1 + m_2) v_f = m_1 v_{o1}$$

$$\boxed{v_{o1} = \left( \frac{m_1 + m_2}{m_1} \right) v_f}$$



$v_f$  ? Mechanical Energy will be conserved

$$\frac{1}{2} (m_1 + m_2) v_f^2 = (m_1 + m_2) g h_f$$

$$\boxed{v_f = \sqrt{2gh_f}}$$

**Speed of bullet= 896 m/s**

# A Collision in Two Dimensions

## Before collision

### x- component of $p_0$

$$p_{0x} = m_1 v_{o1x} + m_2 v_{o2x}$$

$$v_{o1x} = v_{o1} \sin(50^\circ)$$

$$v_{o2x} = v_{o2}$$

$$p_{0x} = m_1 v_{o1} \sin(50^\circ) + m_2 v_{o2}$$

$$\boxed{0.244 \text{ kg m/s}}$$

### y- component of $p_0$

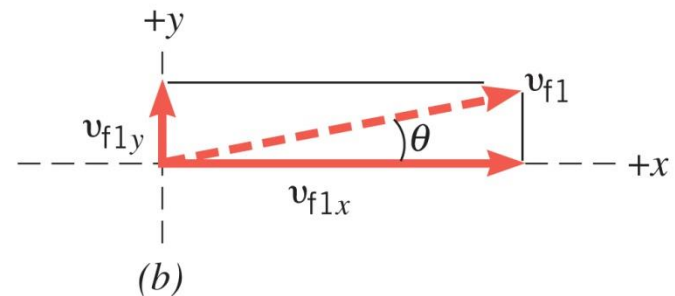
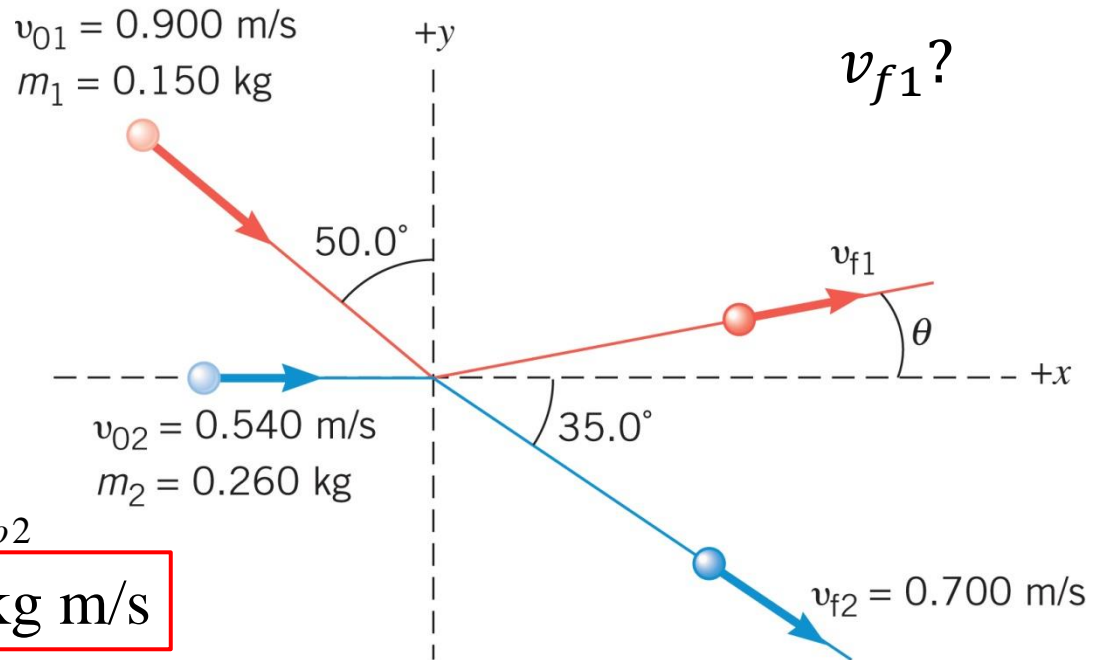
$$p_{0y} = m_1 v_{o1y} + m_2 v_{o2y}$$

$$v_{o1y} = -v_{o1} \cos(50^\circ)$$

$$v_{o2y} = 0$$

$$p_{0y} = -m_1 v_{o1} \cos(50^\circ)$$

$$\boxed{-0.087 \text{ kg m/s}}$$



# After collision

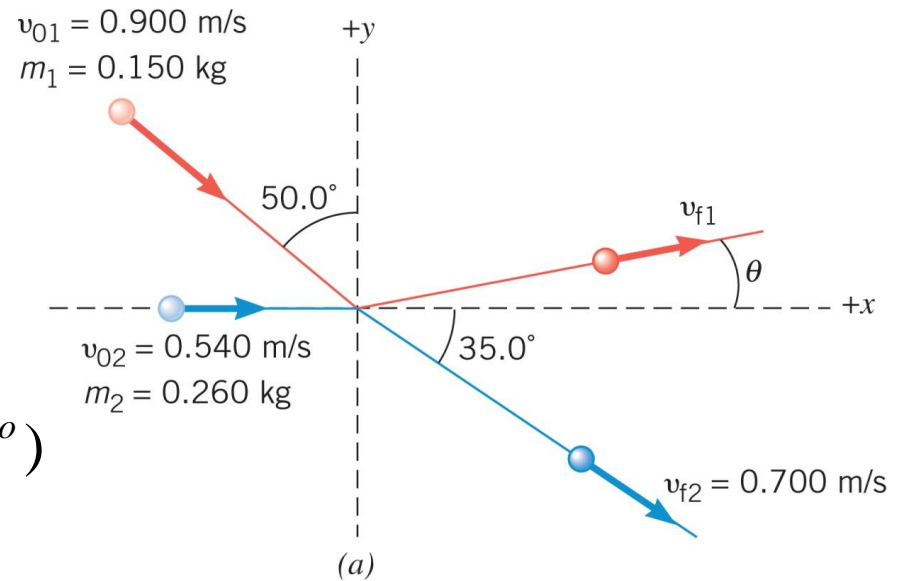
## x- component

$$p_{fx} = m_1 v_{f1x} + m_2 v_{f2x}$$

$$v_{f1x} = v_{f1} \cos \theta$$

$$v_{f2x} = v_{f2} \cos(35^\circ)$$

$$p_{fx} = m_1 v_{f1} \cos \theta + m_2 v_{f2} \cos(35^\circ)$$



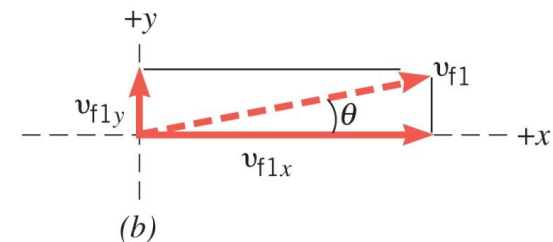
## y- component

$$p_{fy} = m_1 v_{f1y} + m_2 v_{f2y}$$

$$v_{f1y} = v_{f1} \sin \theta$$

$$v_{f2y} = -v_{f2} \sin(35^\circ)$$

$$p_{fy} = m_1 v_{f1} \sin \theta - m_2 v_{f2} \sin(35^\circ)$$



## 7.4 Collisions in Two Dimensions

### Conservation of linear momentum

$$m_1 v_{f1x} + m_2 v_{f2x} = m_1 v_{o1x} + m_2 v_{o2x}$$

$$m_1 v_{f1y} + m_2 v_{f2y} = m_1 v_{o1y} + m_2 v_{o2y}$$

$$p_{fx} = p_{ox} \text{ And } p_{fy} = p_{oy}$$



$$m_1 v_{f1} \cos \theta + m_2 v_{f2} \cos(35^\circ) = 0.244$$

$$v_{f1} \cos \theta = 0.63 \text{ m/s}$$

And

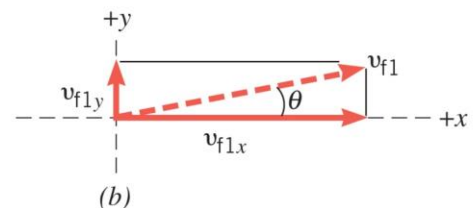
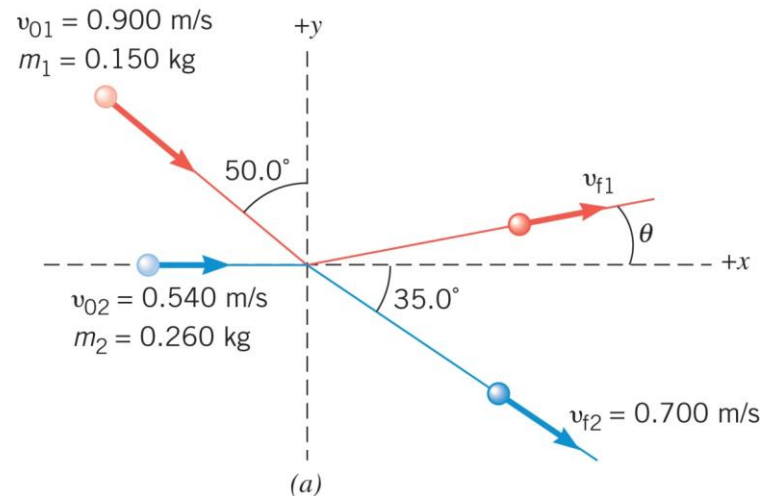
$$m_1 v_{f1} \sin \theta - m_2 v_{f2} \sin(35^\circ) = -0.087$$

$$v_{f1} \sin \theta = 0.116 \text{ m/s}$$

$$v_{f1} = 0.64 \text{ m/s}$$

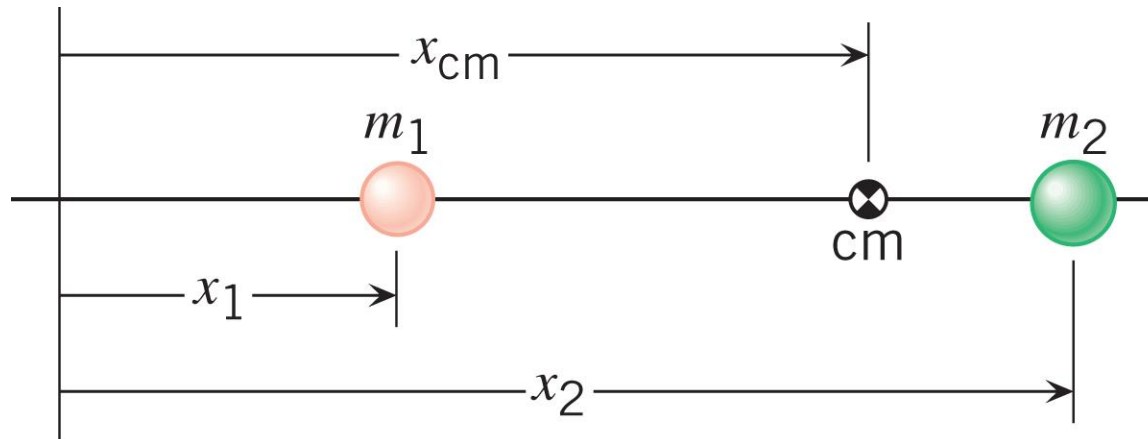
And

$$\theta = 10.4^\circ$$



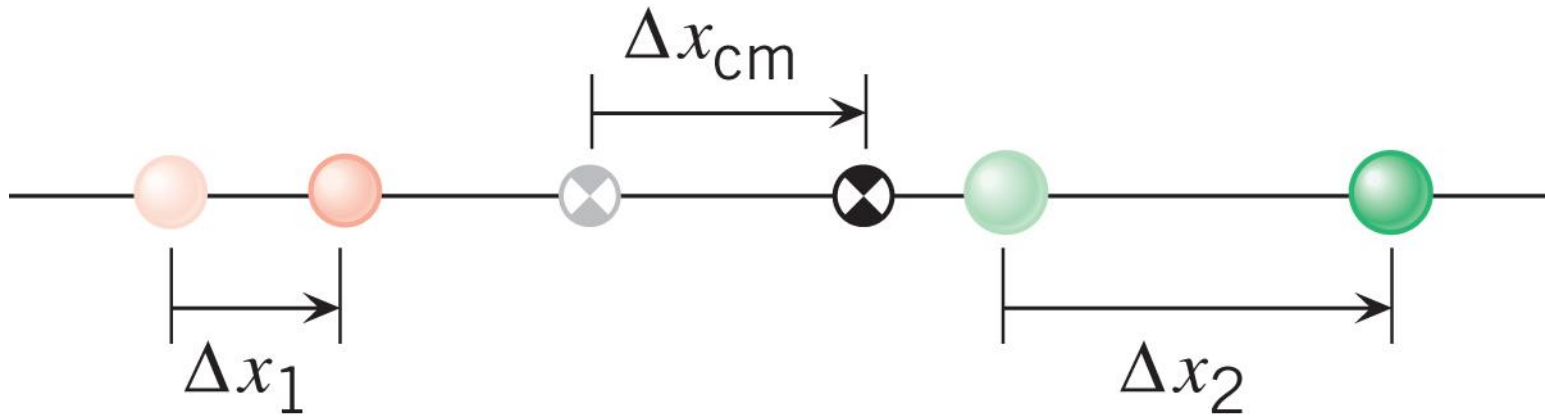
## Center of Mass

The center of mass is a point that represents the average location for the total mass of a system.



$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

## Velocity of center of mass



$$\Delta x_{cm} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2}$$



$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

$$v_{cm} = \frac{p_1 + p_2}{m_1 + m_2}$$

## Question:

A ball hits the floor with a speed  $v$ . It bounces back with exact same speed. Which of the following is true?

- a) The velocity of the ball is conserved.
- b) Both linear momentum and kinetic energy of the ball are conserved.
- c) Only linear momentum of the ball is conserved.
- d) Only kinetic energy of the ball is conserved.

## Question:

A car and a bicycle moving side by side with same velocity. Which of the followings is true

- a) Both have same momentum.
- b) Both require equal impulse to bring them to halt.
- c) Car needs larger impulse to bring it to halt.
- d) None of the above.

## Question:

A collision is called elastic if

- a) The final velocity is zero.
- b) The initial velocity is zero.
- c) Both linear momentum and kinetic energy are not conserved.
- d) Both linear momentum and kinetic energy are conserved.

## Problem:

A 4000 kg truck traveling due North with a speed of 4.0 m/s collides head-on with a 1200 kg car standing on the road. Two vehicles stick together after the collision. What is their velocity just after the collision (magnitude and direction)?

Before Collision

$$p_0 = (m_T v_T + m_C v_C)$$

$$p_f = p_0$$

After Collision

$$p_f = (m_T + m_C)v$$

$$m_T v_T = (m_T + m_C)v$$
$$v = \frac{m_T}{(m_T + m_C)} v_T$$

3.1 m/s North

# Summary of Ch. 7

- Impulse  $\vec{\mathbf{J}} = \vec{\mathbf{F}} \Delta t$

- Linear Momentum

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$$

$$\text{kg} \frac{\text{m}}{\text{s}} \quad \text{OR} \quad \text{N} \cdot \text{s}$$

- Impulse-Momentum Theorem

$$\vec{\mathbf{J}} = \vec{\Delta \mathbf{p}}$$

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

- Principle of Conservation of Linear Momentum

The total linear momentum of an isolated system is constant (conserved).

$$\vec{\mathbf{p}}_f = \vec{\mathbf{p}}_o$$

- 1-D

$$m_1 v_{f1} + m_2 v_{f2} = m_1 v_{o1} + m_2 v_{o2}$$

- 2-D

$$m_1 v_{f1y} + m_2 v_{f2y} = m_1 v_{o1y} + m_2 v_{o2y}$$

$$m_1 v_{f1x} + m_2 v_{f2x} = m_1 v_{o1x} + m_2 v_{o2x}$$

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{p_1 + p_2}{m_1 + m_2}$$

For Recitation Practice

**Chapter 7**

**FOC:** 1, 2, 10 & 15.

**Problems:** 4, 11, 12, 19 & 64.