Chapter 10

Simple Harmonic Motion and Elasticity
### Hooke’s Law: Restoring Force of an Ideal Spring

**Equation:**

\[ F_x = -kx \]

**Units:** N/m

**Explanation:**

- **Equation:** $F_x = kx$
  - $F_x$: Applied force
  - $k$: Spring constant
  - $x$: Displacement

**Units:**

- **Displacement ($x$):** meters (m)
- **Spring constant ($k$):** newtons per meter (N/m)
- **Applied force ($F_x$):** newtons (N)

**Graphical Illustration:**

- Illustration of a spring being stretched or compressed by an applied force.
- The force is shown to be proportional to the displacement, with a negative sign indicating a restoring force.

**Equation for Spring Force:**

\[ F_{Spring} = -kx \]
Example: Tire Pressure Gauge

The spring constant of the spring is 320 N/m and the bar indicator extends 2.0 cm. What force does the air in the tire apply to the spring?

\[ F_{\text{Applied}} = k \times x \]

6.4 N
Conceptual Example: Are Shorter Springs Stiffer?
A 10-coil spring has a spring constant $k$. If the spring is cut in half, so there are two 5-coil springs, what is the spring constant of each of the smaller springs?

$2k$
Hooke’s Law: Restoring Force of an Ideal Spring

The restoring force on an ideal spring is

\[ F_x = -kx \]
10.2 Simple Harmonic Motion and the Reference Circle

**DISPLACEMENT**

\[ x = A \cos \theta = A \cos \omega t \]

\( x \) varies from 0 to ±A
10.2 Simple Harmonic Motion and the Reference Circle

**Amplitude** $A$  the maximum displacement (m)

**Period** $T$  the time required to complete one cycle (s)

**Frequency** $f$  the number of cycles per second (Hz)

**Angular frequency**  $\omega = 2\pi f = \frac{2\pi}{T}$  rad/s

$$f = \frac{1}{T}$$
### VELOCITY

- \( \theta = \omega t \)
- Radius = A

\[
v_x = -v_T \sin \theta = -A \omega \sin \omega t
\]

### ACCELERATION

\[
a_x = -a_c \cos \theta = -A \omega^2 \cos \omega t
\]

\[
= -\omega^2 x
\]
**Example: The Maximum Speed of a Loudspeaker Diaphragm**

The frequency of motion is 1.0 KHz and the amplitude is 0.20 mm.

(a) What is the maximum speed of the diaphragm?

(b) Where in the motion does this maximum speed occur?

\[ v_x = -v_T \sin \theta = -A\omega \sin \omega t \]
\[ v_{\text{max}} = A\omega \]

\[ \omega = 2\pi f \quad \text{[6283.2 rad/s]} \]

\[ v_{\text{max}} = ? \quad \text{[1.27 m/s]} \]

Where?

\[ At \ \theta = 0 \Rightarrow x = 0 \]
For a mass, $m$ attached to a spring and set in vibration on frictionless surface

**FREQUENCY OF VIBRATION**

$$x = A \cos \omega t$$

$$a_x = -A \omega^2 \cos \omega t$$

$$\sum F = ma_x$$

$$\sum F = -kx$$

$$-kx = ma_x$$

$$-kA = -mA \omega^2$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$
\[ W_{\text{elastic}} = \left(F \cos \theta \right) s = \frac{1}{2} \left(kx_o + kx_f \right) \cos(0^\circ) \left(x_f - x_o \right) \]

\[ W_{\text{elastic}} = \frac{1}{2} kx_f^2 - \frac{1}{2} kx_o^2 \]

Position when spring is unstrained

\( x_0 \)

\( x_f \)
DEFINITION OF ELASTIC POTENTIAL ENERGY
The elastic potential energy is the energy that a spring has by virtue of being stretched or compressed. For an ideal spring, the elastic potential energy is

$$PE_{\text{elastic}} = \frac{1}{2} kx^2$$

SI Unit?

joule (J)
**Example: Adding a Mass to a Simple Harmonic Oscillator**

A 0.20-kg ball is attached to a vertical spring. The spring constant is 28 N/m. When released from rest, how far does the ball fall before being brought to a momentary stop by the spring?

\[ kx = mg \]

\[ x = 0.07 \text{ m} \]
10.3 Energy conservation

\[ E = KE_T + KE_R + PE_g + PE_e \]

Principle of Energy Conservation

In absence of any external force

\[ E_f = E_o \]

\[ \frac{1}{2} mv_f^2 + \frac{1}{2} I\omega_f^2 + mgh_f + \frac{1}{2} kx_f^2 = \frac{1}{2} mv_o^2 + \frac{1}{2} I\omega_o^2 + mgh_o + \frac{1}{2} kx_o^2 \]
Simple Pendulum

\[ \sum \tau = I \alpha \]

\[ \sum \tau = -mgx \]

\[ -mgx = mL^2 \frac{a_T}{L} \]

\[ \omega = \sqrt{\frac{g}{L}} \]

\[ T = \frac{1}{f} \]

\[ T = 2\pi \sqrt{\frac{L}{g}} \]

Period of a simple pendulum is independent of mass and Amplitude of the pendulum
Q: A bob attached to a string and set into simple harmonic motion makes a simple pendulum (if angle of swing remains small). One such simple pendulum has a period equals to 0.1 s. Now if the bob is changed to a slightly bigger one with mass double than the previous bob (keeping length of the string same), the period of the simple pendulum will

(a) Become double   (b) Become half   (c) Remain unchanged

Q: A simple pendulum has a period of 6.0 s on the surface of earth. The period of the same pendulum on the surface of moon (where the acceleration due to gravity is 1/6 of that on the surface of earth) will

(a) Be the same   (b) Increase   (c) Decrease

\[ T = 2\pi \sqrt{\frac{L}{g}} \]
**Problem:** A simple pendulum has a ball of mass m attached to a string of length 1.50 m. The ball is pulled to one side through a small angle and then released from rest.

(a) After the ball is released how much time is elapsed before it gains its maximum speed?

(a) After the ball is released how much time is elapsed before it gains its maximum acceleration?

(a) What will be speed of the ball at the time when the acceleration is maximum?

(d) What is the angular frequency of this simple pendulum?

(e) What is the linear frequency of this simple pendulum?

**Simple pendulum:**

\[ \omega = \sqrt{\frac{g}{L}} \quad \text{T} = 2\pi \sqrt{\frac{L}{g}} \quad \text{f (Hz)} = \frac{1}{T} \]
Young’s modulus has the units of pressure: $\text{N/m}^2$

Table 10.1: Young Modulus of Elasticity for some materials
The shear modulus has the units of pressure: $\text{N/m}^2$

Table 10.2: Shear Modulus of Elasticity for some materials
Example 14  J-E-L-L-O
You push tangentially across the top surface with a force of 0.45 N. The top surface moves a distance of 6.0 mm relative to the bottom surface. What is the shear modulus of Jell-O?

\[
F = S \left( \frac{\Delta x}{L_0} \right) A
\]

\[
S = \frac{F L_0}{A \Delta x}
\]

\[
L_0 = 0.03 \text{ m} \quad A = 0.07 \times 0.07 \text{ m}^2 \quad \Delta x = 0.006 \text{ m}
\]

459 N/m$^2$
The Bulk modulus has the units of pressure: N/m²

Table 10.3: Bulk Modulus of Elasticity for some materials
### 10.7 Elastic Deformation

#### Table 10.1  Values for the Young’s Modulus of Solid Materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Young’s Modulus $Y$ (N/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>$6.9 \times 10^{10}$</td>
</tr>
<tr>
<td>Bone</td>
<td></td>
</tr>
<tr>
<td>Compression</td>
<td>$9.4 \times 10^{9}$</td>
</tr>
<tr>
<td>Tension</td>
<td>$1.6 \times 10^{10}$</td>
</tr>
<tr>
<td>Brass</td>
<td>$9.0 \times 10^{10}$</td>
</tr>
<tr>
<td>Brick</td>
<td>$1.4 \times 10^{10}$</td>
</tr>
<tr>
<td>Copper</td>
<td>$1.1 \times 10^{11}$</td>
</tr>
<tr>
<td>Mohair</td>
<td>$2.9 \times 10^{9}$</td>
</tr>
<tr>
<td>Nylon</td>
<td>$3.7 \times 10^{9}$</td>
</tr>
<tr>
<td>Pyrex glass</td>
<td>$6.2 \times 10^{10}$</td>
</tr>
<tr>
<td>Steel</td>
<td>$2.0 \times 10^{11}$</td>
</tr>
<tr>
<td>Teflon</td>
<td>$3.7 \times 10^{8}$</td>
</tr>
<tr>
<td>Titanium</td>
<td>$1.2 \times 10^{11}$</td>
</tr>
<tr>
<td>Tungsten</td>
<td>$3.6 \times 10^{11}$</td>
</tr>
</tbody>
</table>

#### Table 10.2  Values for the Shear Modulus of Solid Materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Shear Modulus $S$ (N/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>$2.4 \times 10^{10}$</td>
</tr>
<tr>
<td>Bone</td>
<td>$1.2 \times 10^{10}$</td>
</tr>
<tr>
<td>Brass</td>
<td>$3.5 \times 10^{10}$</td>
</tr>
<tr>
<td>Copper</td>
<td>$4.2 \times 10^{10}$</td>
</tr>
<tr>
<td>Lead</td>
<td>$5.4 \times 10^{9}$</td>
</tr>
<tr>
<td>Nickel</td>
<td>$7.3 \times 10^{9}$</td>
</tr>
<tr>
<td>Steel</td>
<td>$8.1 \times 10^{10}$</td>
</tr>
<tr>
<td>Tungsten</td>
<td>$1.5 \times 10^{11}$</td>
</tr>
</tbody>
</table>

#### Table 10.3  Values for the Bulk Modulus of Solid and Liquid Materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Bulk Modulus $B$ [N/m² (=Pa)]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Solids</strong></td>
<td></td>
</tr>
<tr>
<td>Aluminum</td>
<td>$7.1 \times 10^{10}$</td>
</tr>
<tr>
<td>Brass</td>
<td>$6.7 \times 10^{10}$</td>
</tr>
<tr>
<td>Copper</td>
<td>$1.3 \times 10^{11}$</td>
</tr>
<tr>
<td>Diamond</td>
<td>$4.43 \times 10^{11}$</td>
</tr>
<tr>
<td>Lead</td>
<td>$4.2 \times 10^{10}$</td>
</tr>
<tr>
<td>Nylon</td>
<td>$6.1 \times 10^{9}$</td>
</tr>
<tr>
<td>Osmium</td>
<td>$4.62 \times 10^{11}$</td>
</tr>
<tr>
<td>Pyrex glass</td>
<td>$2.6 \times 10^{10}$</td>
</tr>
<tr>
<td>Steel</td>
<td>$1.4 \times 10^{11}$</td>
</tr>
<tr>
<td><strong>Liquids</strong></td>
<td></td>
</tr>
<tr>
<td>Ethanol</td>
<td>$8.9 \times 10^{8}$</td>
</tr>
<tr>
<td>Oil</td>
<td>$1.7 \times 10^{9}$</td>
</tr>
<tr>
<td>Water</td>
<td>$2.2 \times 10^{9}$</td>
</tr>
</tbody>
</table>
In general the quantity \( \frac{F}{A} \) is called the Stress.

The change in the dimension divided by that original is called the Strain:

\[
\frac{\Delta L}{L_o}, \quad \frac{\Delta x}{L_o}, \quad \frac{\Delta V}{V_o}
\]

Hooke’s Law for Stress and Strain

Stress is directly proportional to strain.

\[
\frac{F}{A} = Y\left( \frac{\Delta L}{L_o} \right)
\]

\[
\frac{F}{A} = S\left( \frac{\Delta x}{L_o} \right)
\]

\[
\Delta P = -B\left( \frac{\Delta V}{V_o} \right)
\]

SI Unit of Stress: \( \textbf{N/m}^2 \)  
SI Units of Strain: A unit less quantity
Q: With same applied force rubber can be elastically deformed thousands of time more than steel. Does it mean that the Young’s modulus of elasticity of rubber is

(a) Thousands of time larger than that of steel?

(b) Thousands of time smaller than that of the steel?

(c) Same as that of steel.
For Recitation Practice

Chapter 10

FOC: 2, 3, 4, 11 & 14.

Problems: 5, 11, 17, 27 & 51.