Chapter 17

The Principle of Linear Superposition and Interference Phenomena

- **THE PRINCIPLE OF LINEAR SUPERPOSITION**

  When two or more waves are present simultaneously at the same place, the resultant disturbance is the sum of the disturbances from the individual waves.

- **Constructive Interference**

  When two waves meet such a way that hills (condensations) meet hills and valleys (rarefactions) meet the valleys they said to be ***exactly in phase*** and they exhibit ***constructive interference.***

- **Destructive Interference**

  When two waves meet such a way that hills (condensations) meet valleys (rarefactions) and valleys meet the hills they said to be ***exactly out of phase*** and they exhibit ***destructive interference.***

- **Coherent Sources**

  If the wave patters do not shift relative to one another as time passes, the sources are said to be ***coherent*.**

For constructive interference:

Path difference = $m\,\lambda$

where m is 0, 1, 2, 3, …

For destructive interference:

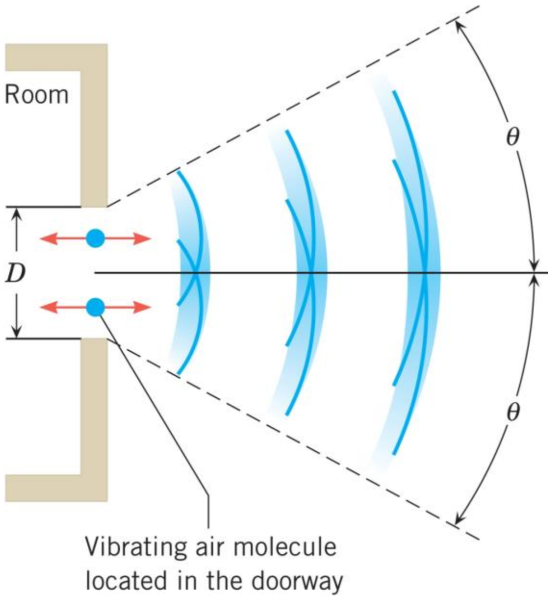
Path difference = $n\,\lambda/2$

where n is 1, 3, 5, 7, …

- **Diffraction**

The bending of a wave around an obstacle or the edges of an opening is called ***diffraction***.

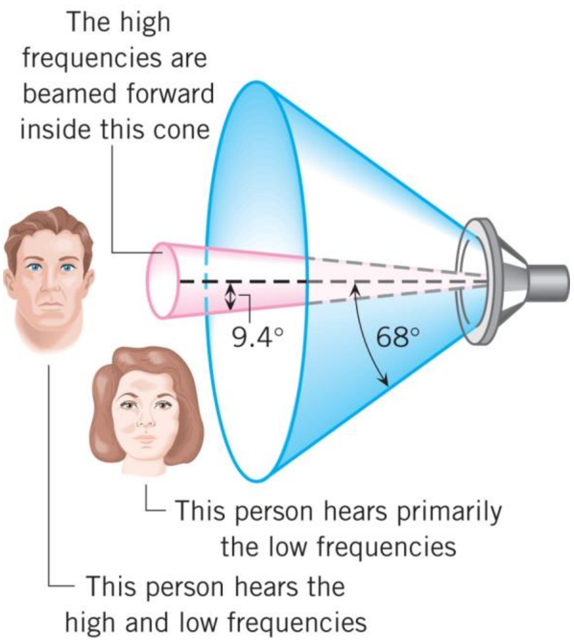
For rectangular opening of width D



single slit – first minimum

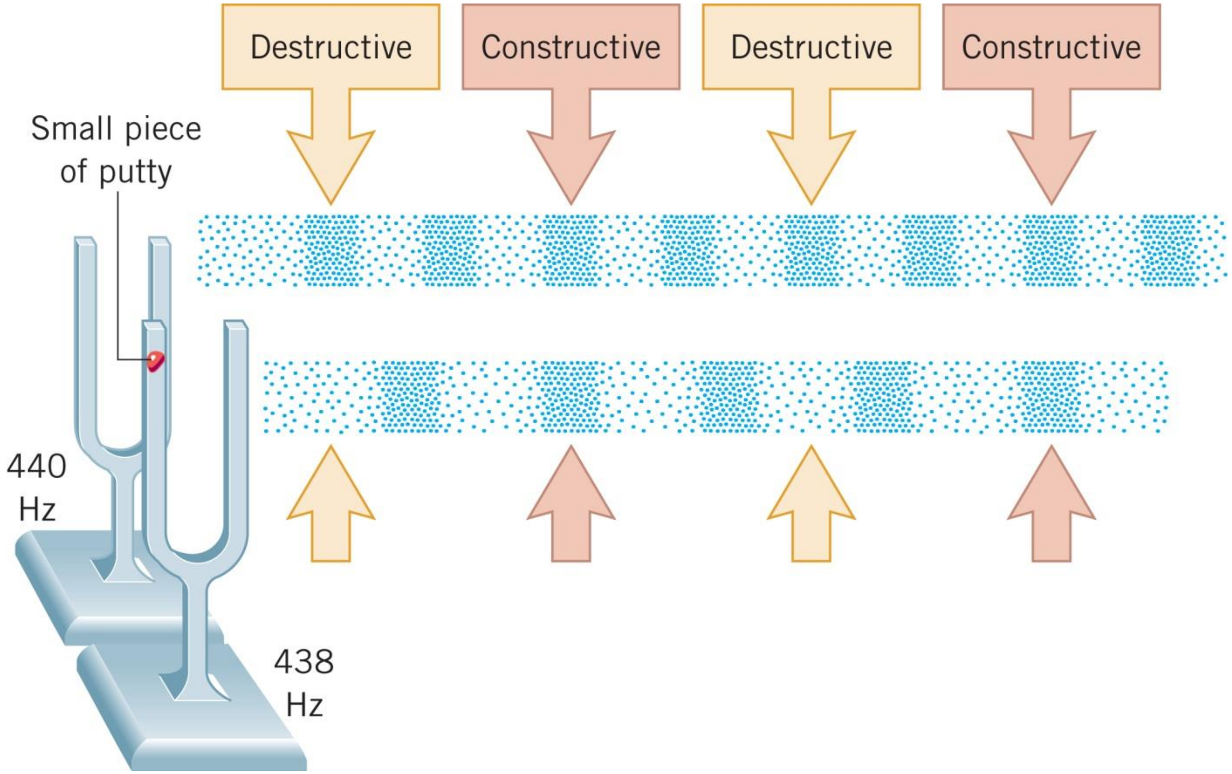
$$\boxed{\sin\theta = \frac{\lambda}{D}}$$

For circular opening of diameter D

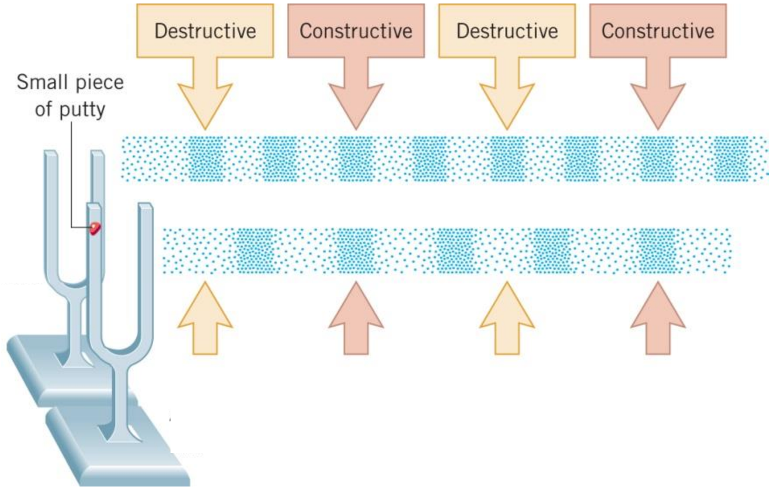


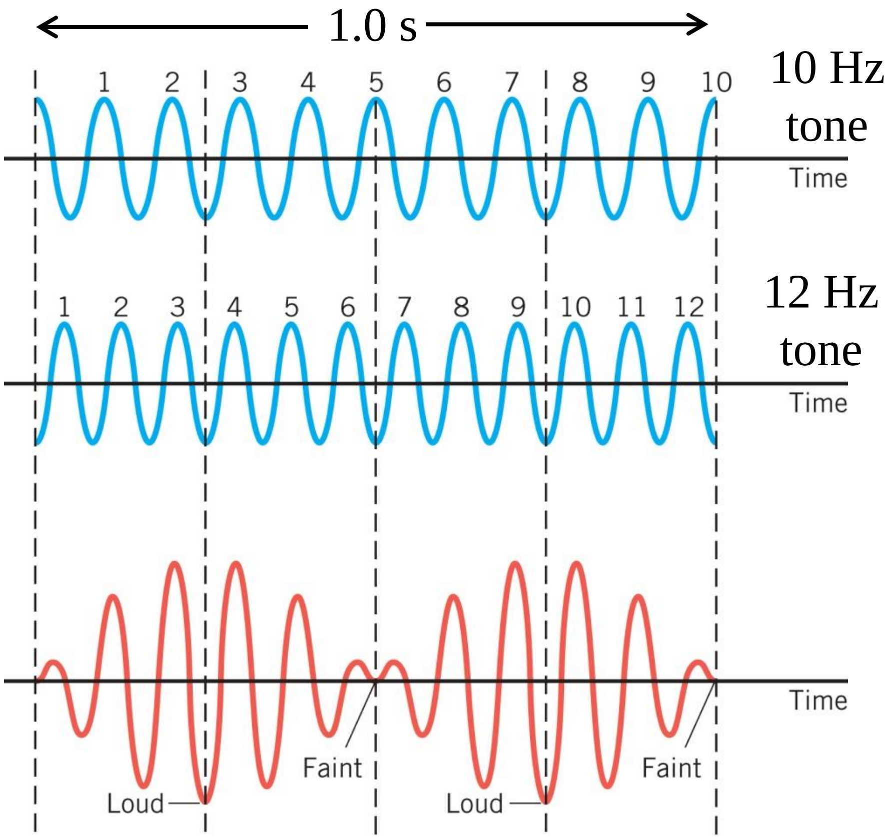
Circular opening – first minimum

$$\boxed{\sin\theta = 1.22\frac{\lambda}{D}}$$



Two overlapping waves with *slightly different frequencies* gives rise to the phenomena of beats.





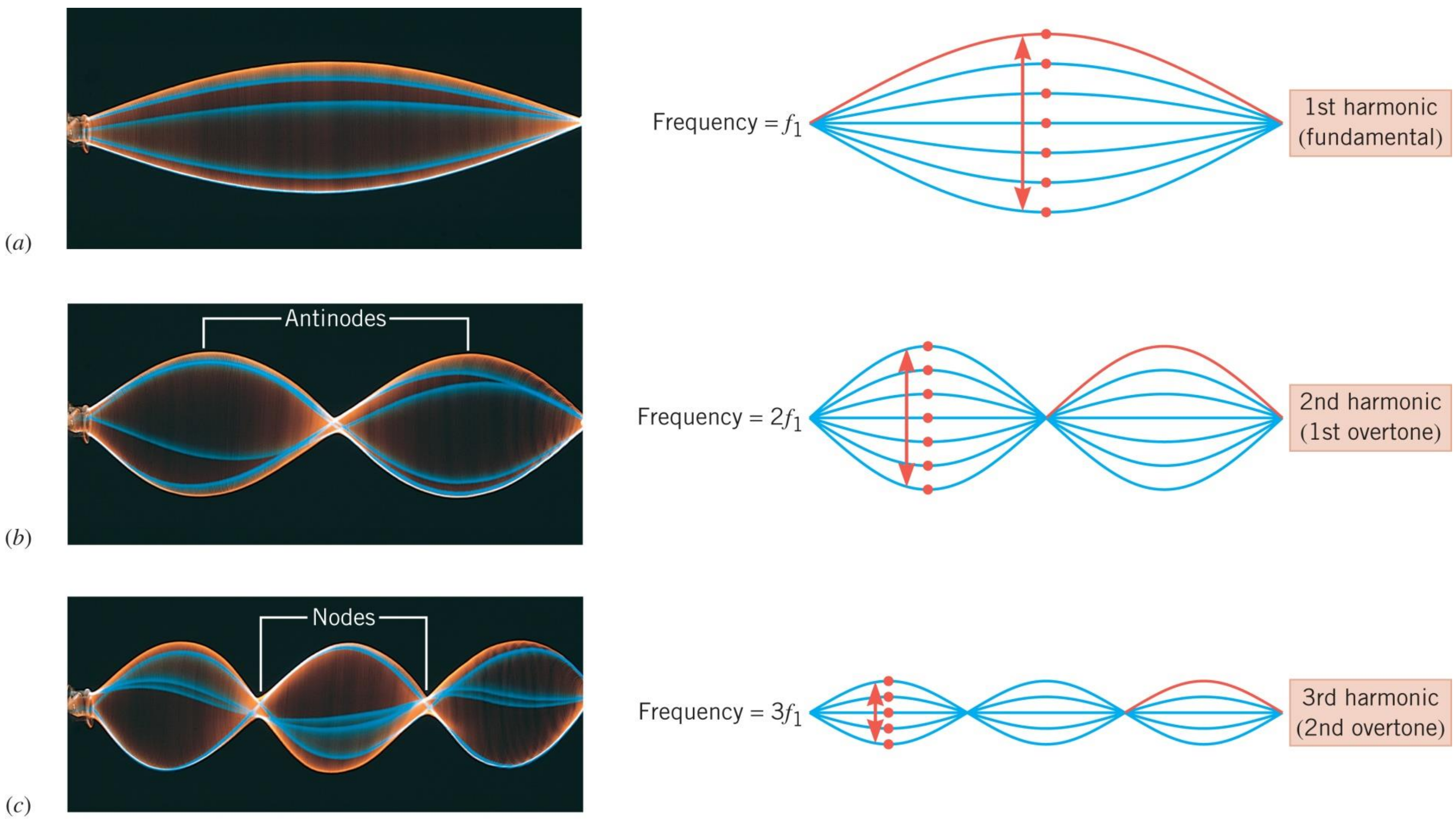
An observer hears the sound loudness rise and fall at the rate of 2 per seconds

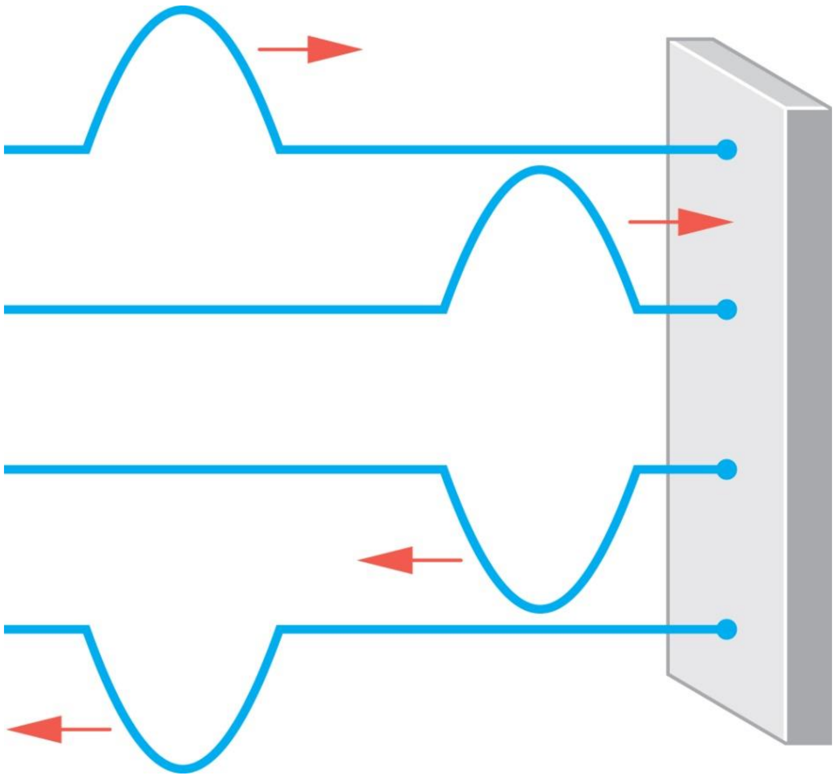
Frequency? $\boxed{2\ \text{Hz}}$

The ***beat frequency*** is the ***difference*** between the two sound frequencies.

17.5 *Transverse Standing Waves*

Transverse standing wave patters





In reflecting from the wall, a forward-traveling half-cycle becomes a backward-traveling half-cycle that is inverted.

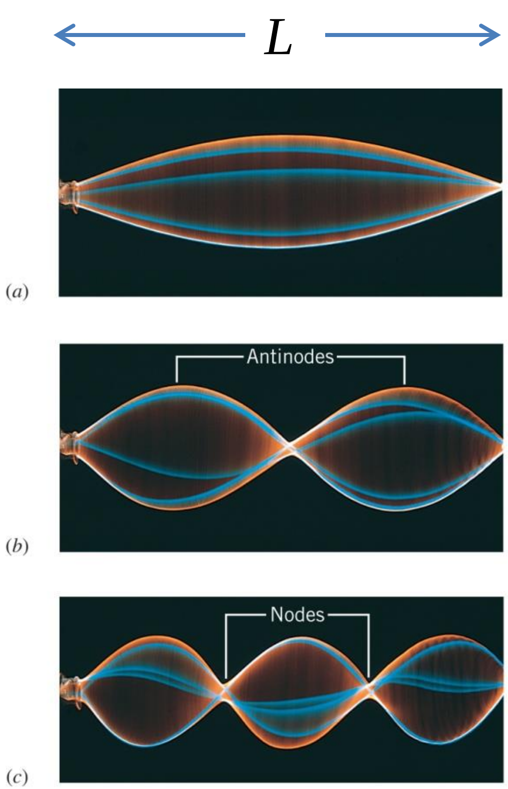
Unless the timing is right, the newly formed and reflected cycles tend to offset one another.

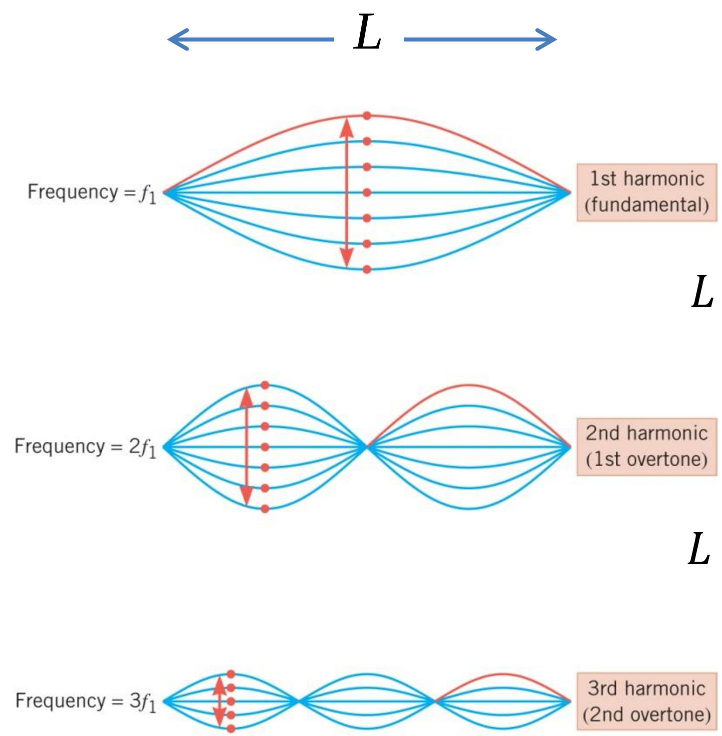
Repeated reinforcement between newly created and reflected cycles causes a large amplitude standing wave to develop.

$n = 1$

$n = 2$

$n = 3$





$$L = \frac{\lambda_1}{2}$$

$$L = 2\frac{\lambda_2}{2}$$

$$L = 3\frac{\lambda_3}{2}$$

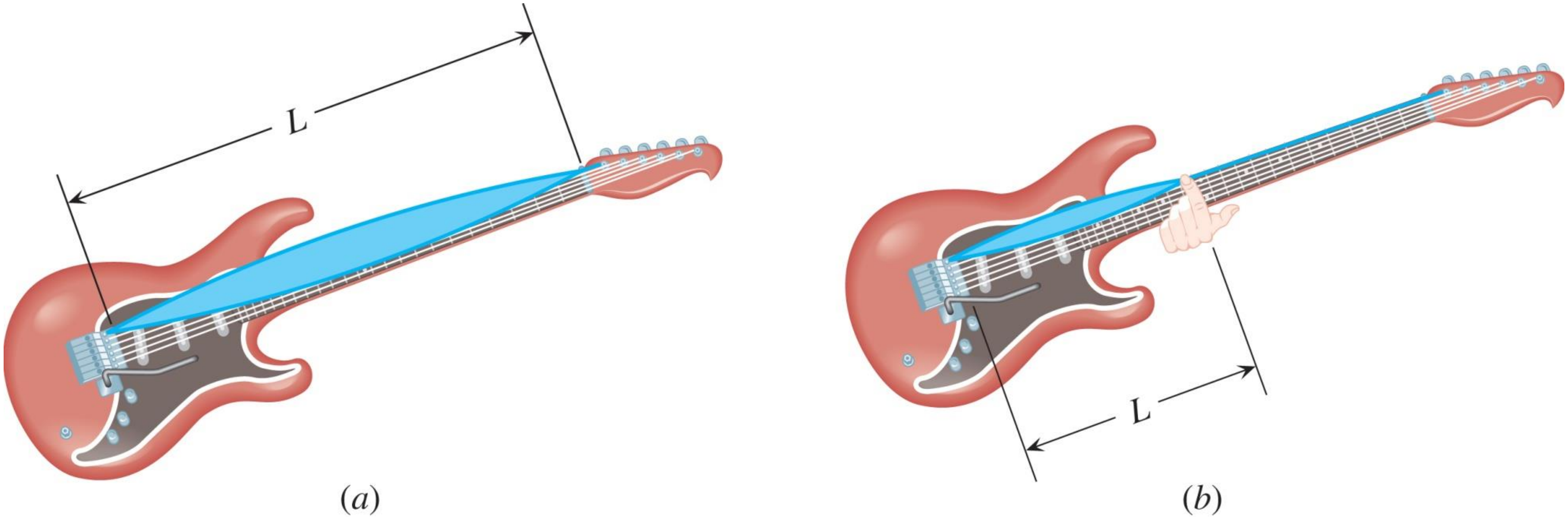
For n number of antinodes $\quad L = n\frac{\lambda_n}{2} \Rightarrow \lambda_n = \frac{2L}{n}$

Since $v = \lambda f$ OR $f = \frac{v}{\lambda}$

String fixed at both ends

$$f_n = n\left(\frac{v}{2L}\right) \qquad n = 1, 2, 3, 4, \ldots$$

$$f_n = nf_1$$



$$f_n = n\left(\frac{v}{2L}\right) \qquad n = 1, 2, 3, 4, \ldots$$

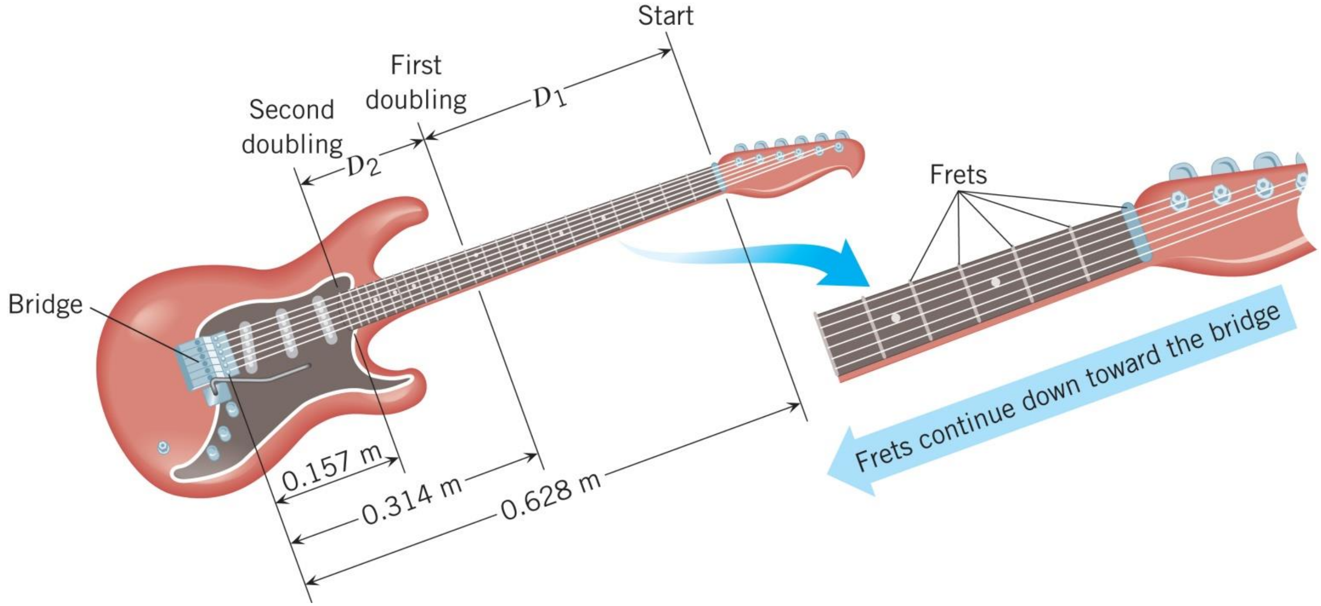
$$f_1 = fundamental\ frequency\ OR\ First\ harmonics = \frac{v}{2L}$$

$$f_n = n f_1$$

Conceptual Example: The Frets on a Guitar

Frets allow a the player to produce a complete sequence of musical notes on a single string. Starting with the fret at the top of the neck, each successive fret shows where the player should press to get the next note in the sequence.

Musicians call the sequence the chromatic scale, and every thirteenth note in it corresponds to one octave, or a doubling of the sound frequency. The spacing between the frets is greatest at the top of the neck and decreases with each additional fret further on down. Why does the spacing decrease going down the neck?

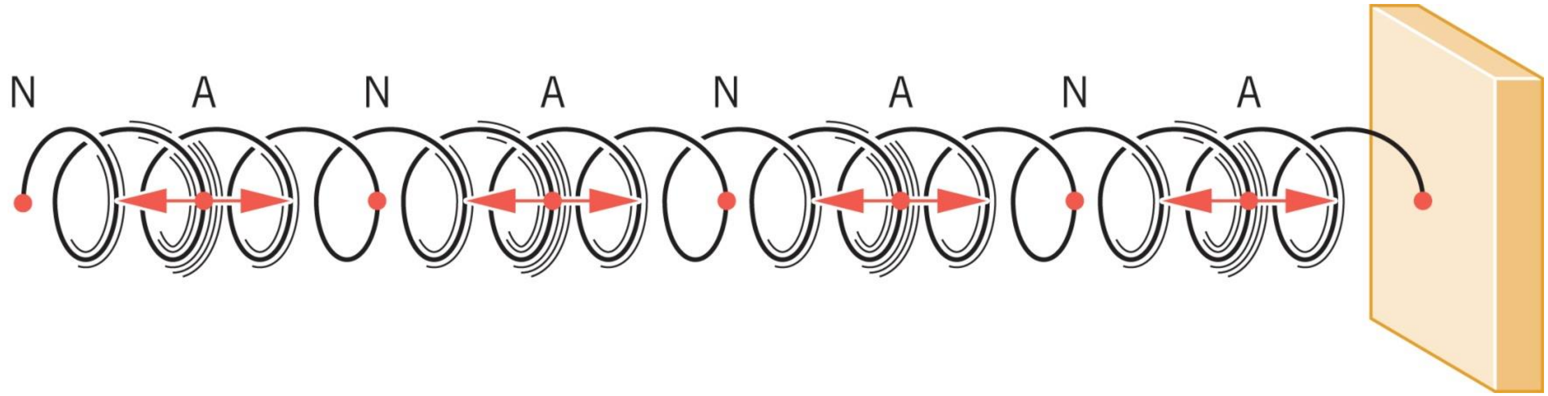


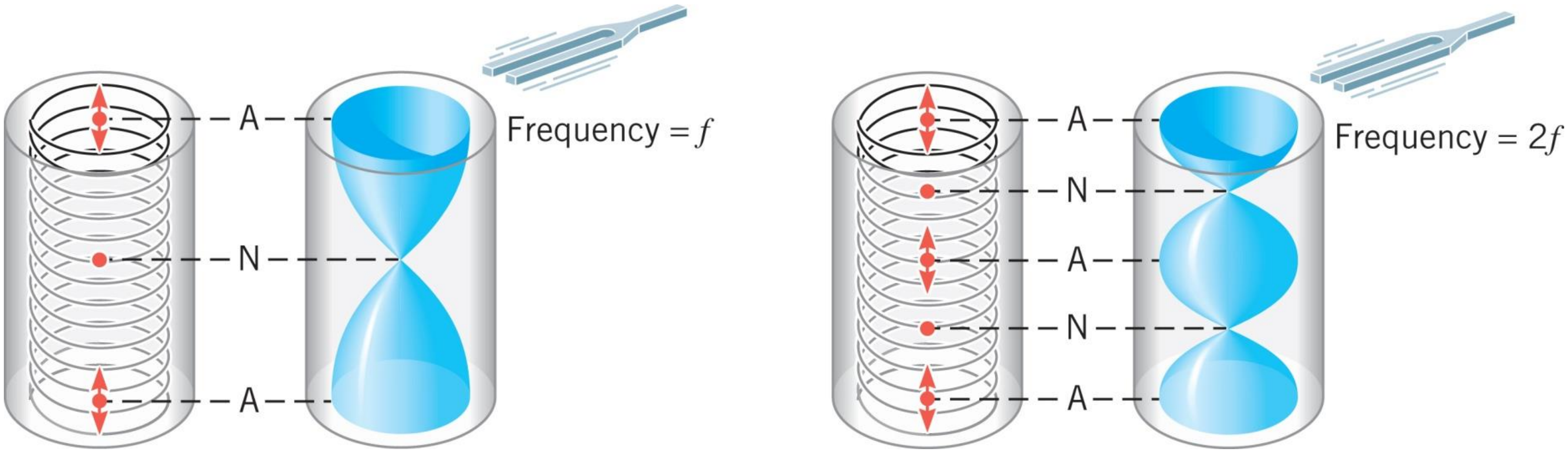
$$f_n = n\left(\frac{v}{2L}\right)$$

$$f_n = n f_1$$

The spacing between the frets is greatest at the top of the neck and decreases with each additional frets further down

A longitudinal standing wave pattern on a slinky.





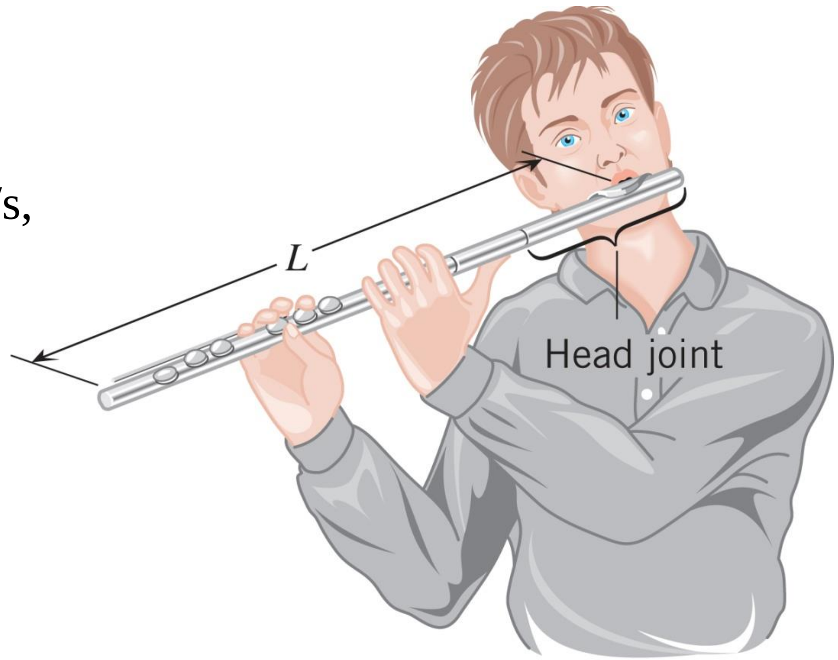
Tube open at both ends

$$f_n = n\left(\frac{v}{2L}\right) \qquad n = 1, 2, 3, 4, \ldots$$

Fixed at both ends or free (open tube) at both ends

Example: Playing a Flute

When all the holes are closed on one type of flute, the lowest note it can sound is middle C (261.6 Hz). If the speed of sound is 343 m/s, and the flute is assumed to be a cylinder open at both ends, determine the distance L.

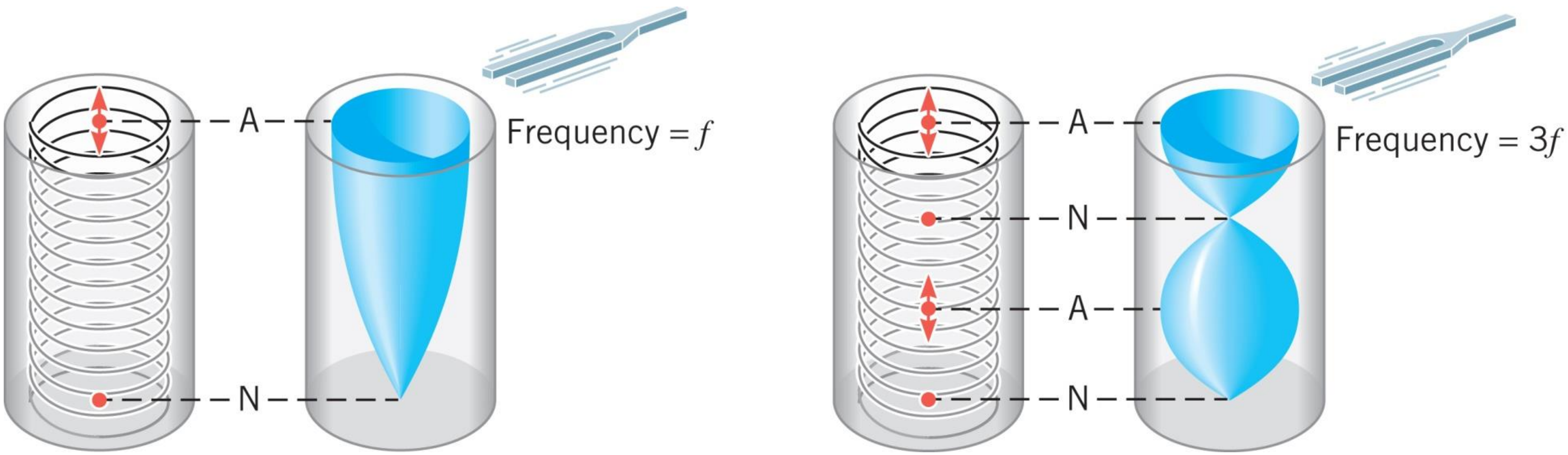


$$f_n = n\left(\frac{v}{2L}\right) \qquad n = 1, 2, 3, 4, \ldots$$

$$L = \frac{nv}{2f_n} = \frac{1(343\,\mathrm{m/s})}{2(261.6\,\mathrm{Hz})}$$

$$\boxed{L = 0.656\,\mathrm{m}}$$

17.6 Longitudinal Standing Waves



Tube open at one end $$f_n = n\left(\frac{v}{4L}\right) \qquad n = 1, 3, 5, \ldots$$

Or string fixed at one end and free at other end

Problem: 45

The fundamental frequencies of two air columns are same. Column A is open at both ends, while column B is open at only one end. The length of column A is 0.70 m. What is the length of column B?

Since the fundamental frequencies of the two air columns are the same

$$f_1^{A} = f_1^{B}$$

$$f_1^{A} = (1)\left(\frac{v}{2L_A}\right) \quad \text{and} \quad f_1^{B} = (1)\left(\frac{v}{4L_B}\right)$$

Therefore

$$\frac{v}{2L_A} = \frac{v}{4L_B}$$

$$L_B = \frac{1}{2}L_A \qquad L_B = \frac{1}{2}(0.70\ m) \qquad \boxed{= 0.35\ \text{m}}$$

Problem: 53

A string is fixed from both ends and is vibrating at 130 Hz, which is it's 3rd harmonic frequency. The linear density of the string is 5.6 x 10^{-3} kg/m, and it is under a tension of 3.3 N. Determine the length of the string.

$$f_3 = 3\left(\frac{v}{2L}\right) \quad \text{OR} \quad L = \frac{3v}{2f_3} \quad \text{and} \quad v = \sqrt{\frac{F}{m/L}}$$

$$L = \frac{3v}{2f_3} = \frac{3}{2f_3}\sqrt{\frac{F}{m/L}}$$

Although L appears on both sides of Equation (2), no further algebra is required. This is because L appears in the ratio m/L on the right side. This ratio is the linear density of the string, which has a known value of 5.6×10−3 kg/m. Therefore, the length of the string is

$$L = \frac{3}{2(130\ Hz)}\sqrt{\frac{3.3\ N}{5.6\times 10^{-3} kg/m}} \qquad \boxed{= 0.28\ \text{m}}$$

Problem: 15

The entrance to a large lecture room consists of two side-by-side doors, one hinged on the left and the other hinged on the right. Each door is 0.700 m wide. Sound of the frequency 607 Hz is coming through the entrance from within the room. The speed of sound is 343 m/s. What is the diffraction angle of the sound after it passes through the doorway when

(a) One door is open and

(b) Both doors are open.

(a) $$\sin\theta = \frac{\lambda}{D} \quad \text{and} \quad v = \lambda f \qquad \sin\theta = \frac{v}{fD}$$

$$\sin\theta = \frac{343\ m/s}{(607\ Hz)(0.70m)}$$

$$\boxed{\theta = 53.8°}$$

(b)

When both doors are open, D = 2 × 0.700 m and the diffraction angle is

$$\sin\theta = \frac{343\ m/s}{(607\ Hz)(2\times 0.70m)}$$

$$\boxed{\theta = 23.8°}$$

For Recitation

Ch. 17

FOC: 2, 3, 5, 14, 16.

Problems: 7, 14, 19, 41 & 43.