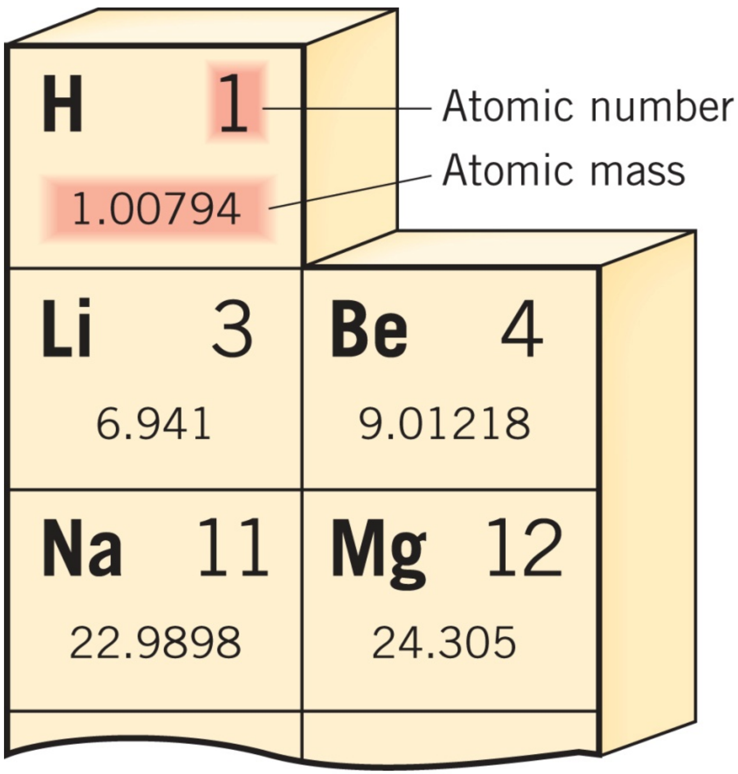
Chapter 14

The Ideal Gas Law and Kinetic Theory

To facilitate comparison of the mass of one atom with another, a mass scale know as the ***atomic mass scale*** has been established.

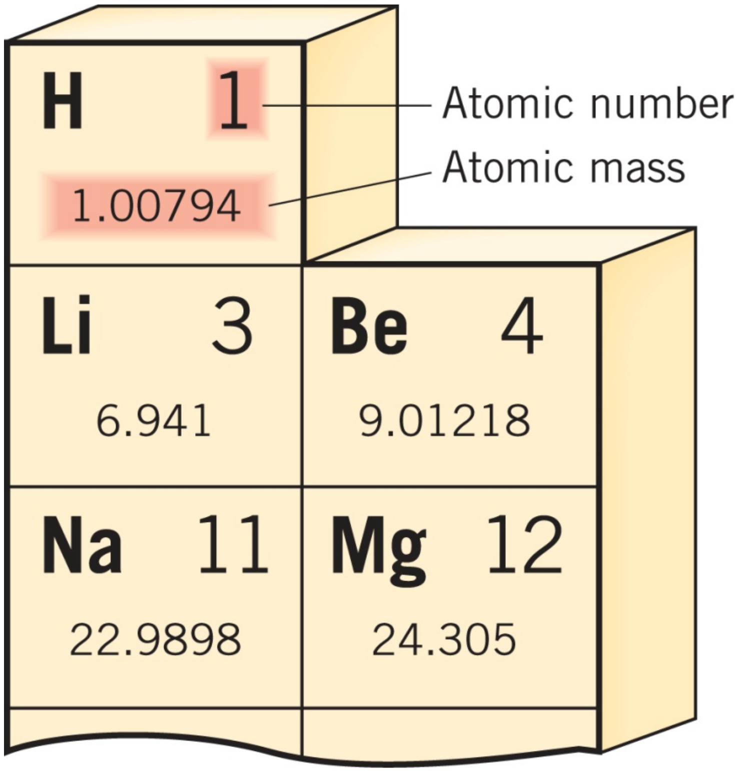
The unit is called the ***atomic mass unit*** (symbol u). The reference element is chosen to be the most abundant isotope of carbon, which is called carbon-12. Carbon-12 has exactly 12 atomic mass units.



$$1\,\text{u} = 1.6605\times10^{-27}\,\text{kg}$$

The atomic mass is given in atomic mass units. For example, a Li atom has a mass of 6.941u.

One gram-***mole*** of a substance contains as many particles as there are atoms in 12 grams of the isotope carbon-12.



The number of atoms per mole is known as **Avogadro's number**, N_A.

$$N_A = 6.022 \times 10^{23}\,\text{mol}^{-1}$$

$$n = \frac{N}{N_A}$$

number of moles

number of atoms

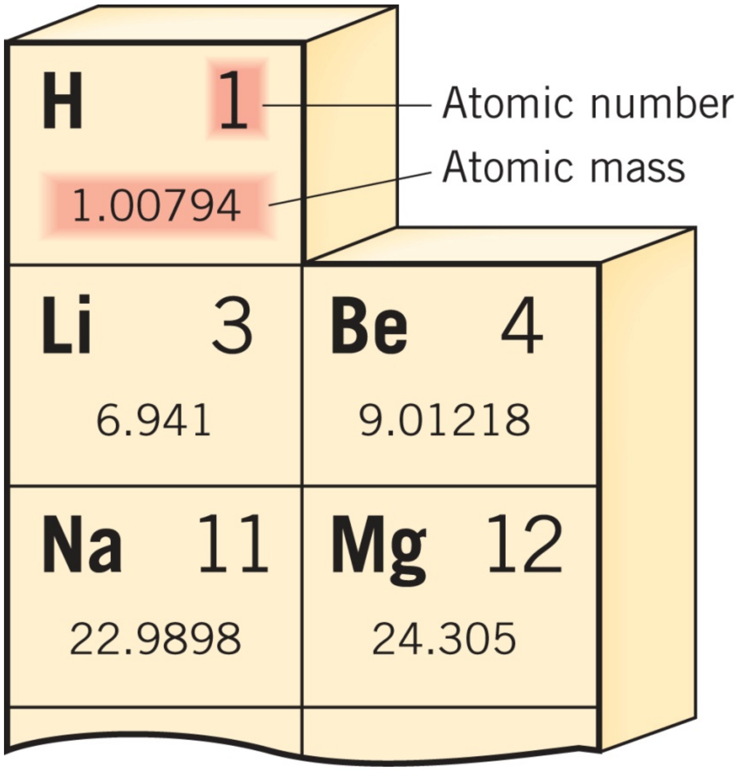
$$n = \frac{N}{N_A}$$

$$n = \frac{m_{\text{particle}} N}{m_{\text{particle}} N_A}$$

$$n = \frac{m}{\text{Mass per mole}}$$

The mass per mole (in g/mol) of a substance has the same numerical value as the atomic or molecular mass of the substance (in atomic mass units).

For example Hydrogen has an atomic mass of 1.00794 g/mol, while the mass of a single hydrogen atom is 1.00794 u.



Example: **The Hope Diamond and the Rosser Reeves Ruby**

The Hope diamond (44.5 carats) is almost pure carbon. The Rosser Reeves ruby (138 carats) is primarily aluminum oxide (Al_2O_3). One carat is equivalent to a mass of 0.200 g. Determine

(a) the number of carbon atoms in the Hope diamond and

(b) the number of Al_2O_3 molecules in the ruby.

(a)

$$n = \frac{m}{\text{Mass per mole}} = \frac{(44.5\text{ carats})[(0.200\text{ g})/(1\text{ carat})]}{12\text{ g/mol}}$$

0.74 moles

$$N = nN_A$$

$$\boldsymbol{N} = (0.74\text{ mol})(6.022\times10^{23}\text{ mol}^{-1})$$

$$N = 4.46\times10^{23}\text{ atoms}$$

(b) the number of Al_2O_3 molecules in the ruby.

$$n = \frac{m}{\text{Mass per mole}} = \frac{(138\text{ carats})[(0.200\text{ g})/(1\text{ carat})]}{\underbrace{101.96}_{2(26.98)+3(15.99)}\ \text{g/mol}}$$

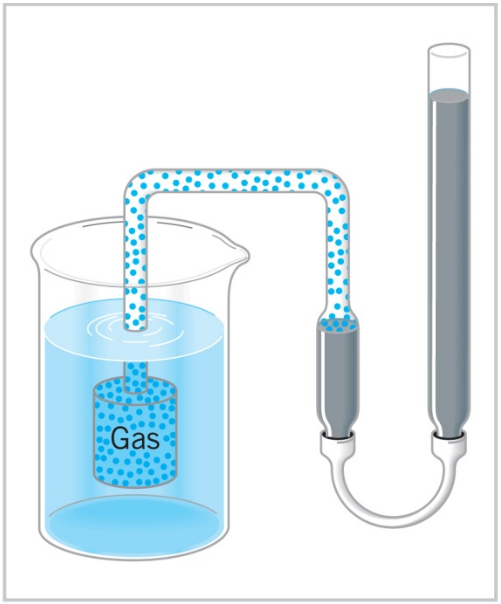
$$\boxed{n = 0.271\text{ moles}}$$

$$N = nN_A$$

$$N = nN_A = (0.271\text{ mol})(6.022\times10^{23}\text{ mol}^{-1})$$

$$\boxed{1.63\times10^{23}\text{ atoms}}$$

An ***ideal gas*** is an idealized model for real gases that have sufficiently low densities.

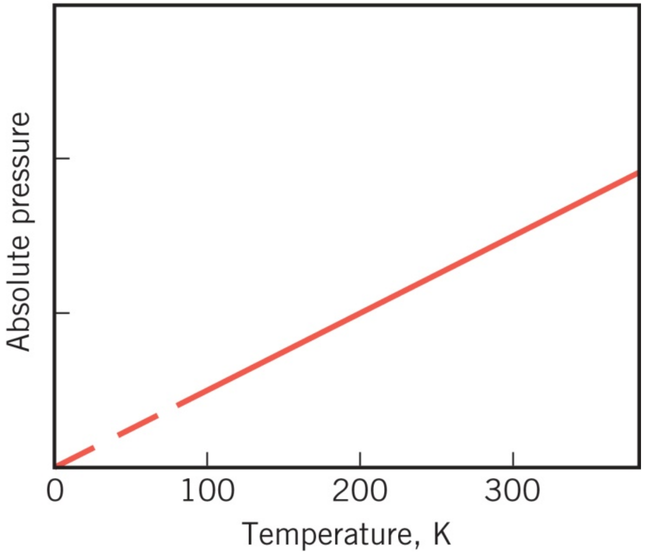


The condition of low density means that the molecules are so far apart that they do not interact except during collisions, which are effectively elastic.

- No interactions
- All collisions are elastic

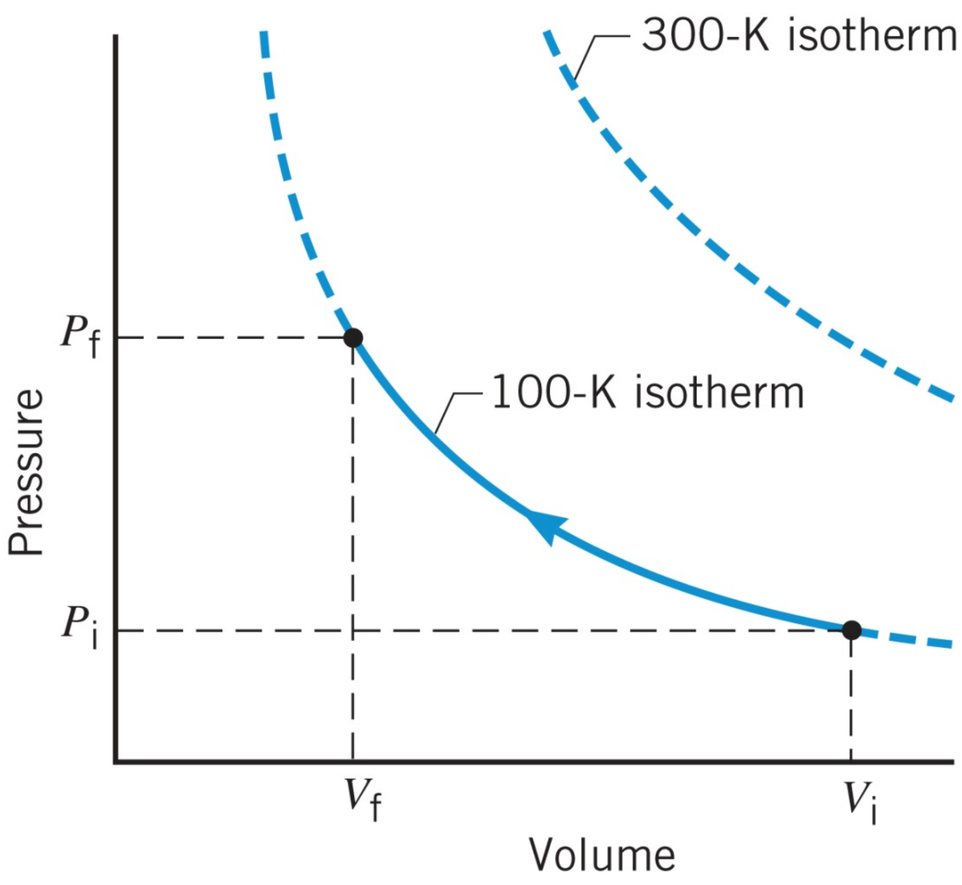
At constant volume the pressure is proportional to the temperature.

$$P \propto T$$



At constant temperature, the pressure is inversely proportional to the volume.

$$P \propto 1/V$$



The pressure is also proportional to the amount of gas.

$$P \propto n$$

$$P \propto \frac{nT}{V}$$

THE IDEAL GAS LAW

The absolute pressure of an ideal gas is directly proportional to the Kelvin temperature and the number of moles of the gas and is inversely proportional to the volume of the gas.

$$P \propto \frac{nT}{V} \quad \Rightarrow \quad P = \frac{nRT}{V}$$

$$\boxed{PV = nRT}$$

$$R = 8.31\,\mathrm{J}/(\mathrm{mol}\cdot\mathrm{K})$$

$$n = \frac{N}{N_A}$$

$$PV = nRT \quad \Rightarrow \quad PV = N\left(\frac{R}{N_A}\right)T$$

$$k = \frac{R}{N_A} = \frac{8.31\,\mathrm{J/(mol \cdot K)}}{6.022 \times 10^{23}\,\mathrm{mol^{-1}}} = 1.38 \times 10^{-23}\,\mathrm{J/K}$$

Boltzmann Constant

$$PV = NkT$$

$$PV = nRT \quad \text{OR} \quad PV = NkT$$

n: No. of moles

N: No. of particles

***Example:* Oxygen in the Lungs**

In the lungs, the respiratory membrane separates tiny sacs of air (pressure $1.00\text{x}10^5$Pa) from the blood in the capillaries. These sacs are called alveoli. The average radius of the alveoli is 0.125 mm, and the air inside contains 14% oxygen. Assuming that the air behaves as an ideal gas at 310 K (body temperature), find the number of oxygen molecules in one of these sacs.

$$PV = NkT$$

$$N = \frac{PV}{kT}$$

$$N = \frac{\left(1.00\times10^{5}\,\text{Pa}\right)\left[\frac{4}{3}\pi\left(0.125\times10^{-3}\,\text{m}\right)^{3}\right]}{\left(1.38\times10^{-23}\,\text{J/K}\right)\left(310\,\text{K}\right)}$$

$$\boxed{1.9\times10^{14}\ \text{molecules of air}}$$

$$\text{Number of molecules of oxygen in one sac} = ?$$

$$\left(1.9\times10^{14}\right)\times\left(0.14\right)$$

$$\boxed{2.7\times10^{13}\ \text{molecules}}$$

Consider a sample of an ideal gas that is taken from an initial to a final state, with the amount of the gas remaining constant.

$$PV = nRT \Longrightarrow \frac{PV}{T} = nR = \text{constant}$$

$$\Downarrow$$

$$\frac{P_f V_f}{T_f} = \frac{P_i V_i}{T_i}$$

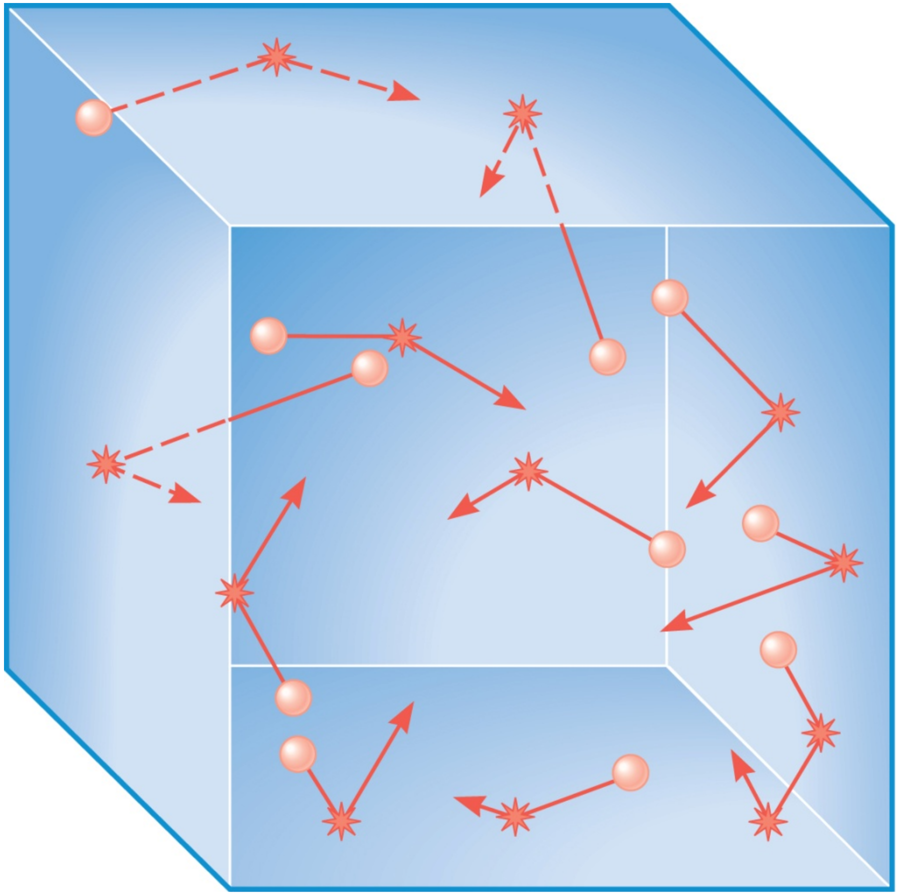
Constant T, constant n: $P_f V_f = P_i V_i$ **Boyle's law**

Constant P, constant n: $\frac{V_f}{T_f} = \frac{V_i}{T_i}$ **Charles' law**

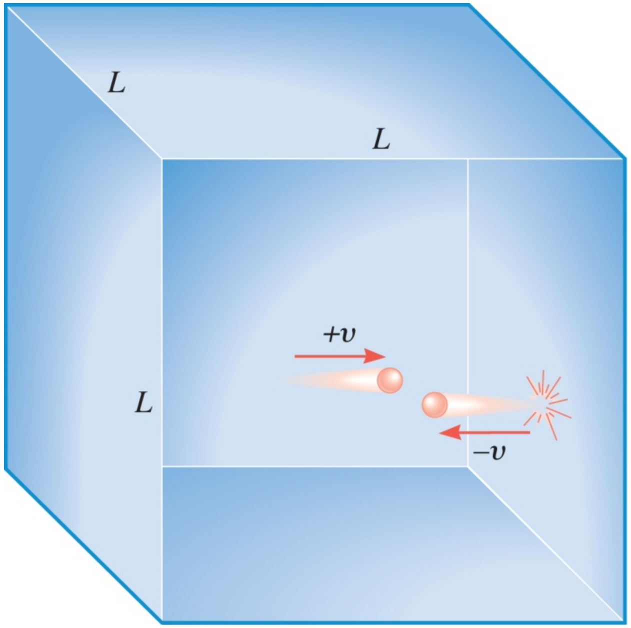
- The particles are in constant, random motion, colliding with each other and with the walls of the container.

- Each collision changes the particle's speed.

- As a result, the atoms and molecules have different speeds.



KINETIC THEORY



$$\sum F = ma \qquad a = \frac{\Delta v}{\Delta t}$$

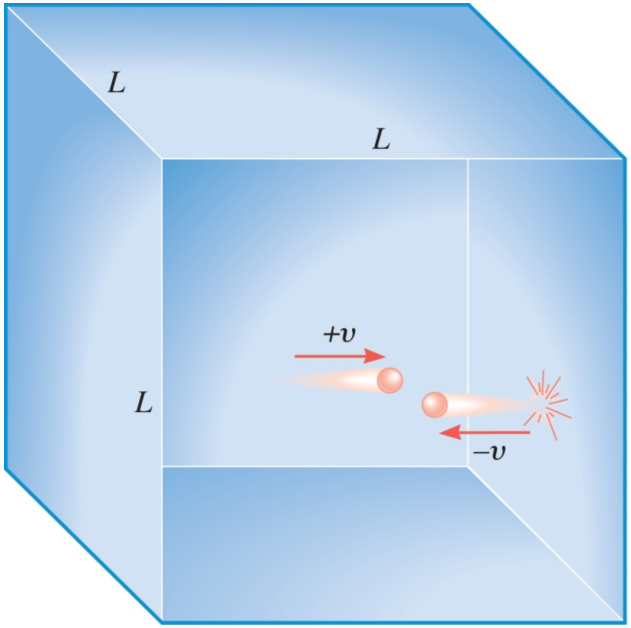
$$\sum F = m\frac{\Delta v}{\Delta t} = \frac{\Delta(mv)}{\Delta t}$$

$$= \frac{\Delta p}{\Delta t}$$

$$\text{Average force} = \frac{\text{Final momentum - Initial momentum}}{\text{Time between successive collisions}}$$

$$\text{Average force} = \frac{(-mv)-(+mv)}{2L/v} = \frac{-mv^2}{L}$$

For a single molecule, the magnitude of average force is:



$$F = \frac{mv^2}{L}$$

For N molecules, the average force is:

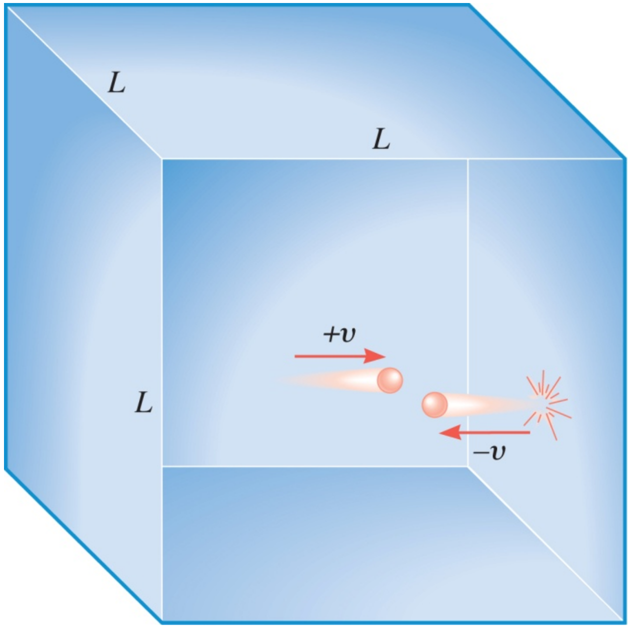
$$F = \left(\frac{N}{3}\right)\left(\frac{m\overline{v^2}}{L}\right)$$

root-mean-square speed

Since $P = \frac{F}{A}$ and $A = L^2$

$$P = \frac{F}{A} = \frac{F}{L^2} = \left(\frac{N}{3}\right)\left(\frac{m\overline{v^2}}{L^3}\right)$$

volume



$$\boldsymbol{P} = \left(\frac{\boldsymbol{N}}{3}\right)\left(\frac{\boldsymbol{m}\overline{\boldsymbol{v}^2}}{\boldsymbol{L}^3}\right)$$

$$P = \left(\frac{N}{3}\right)\left(\frac{m\overline{v^2}}{V}\right)$$

$\Downarrow$

NkT $\qquad$ $\overline{\mathrm{KE}}$

$$PV = \tfrac{1}{3}N\left(mv_{rms}^2\right) = \tfrac{2}{3}N\left(\tfrac{1}{2}mv_{rms}^2\right)$$

$$NkT = \frac{2}{3}N(\overline{KE}) \quad \Rightarrow \quad (\overline{KE}) = \frac{3}{2}kT$$

$$\boxed{\overline{\mathrm{KE}} = \tfrac{1}{2}mv_{rms}^2 = \tfrac{3}{2}kT}$$

THE INTERNAL ENERGY OF A MONATOMIC IDEAL GAS

$$\overline{\mathrm{KE}} = \tfrac{1}{2} m v_{rms}^{2} = \tfrac{3}{2} kT$$

$$U = N\,\tfrac{3}{2} kT = \tfrac{3}{2} nRT$$

Example: The Speed of Molecules in Air

Air is primarily a mixture of nitrogen N_2 molecules (molecular mass 28.0 u) and oxygen O_2 molecules (molecular mass 32.0 u). Assume that each behaves as an ideal gas and determine the rms speeds of the nitrogen and oxygen molecules when the temperature of the air is 293 K.

$$\tfrac{1}{2} m v_{rms}^2 = \tfrac{3}{2} kT \quad \Rightarrow \quad v_{rms} = \sqrt{\frac{3kT}{m}}$$

For Nitrogen

$$\boldsymbol{m} = \frac{28.0\,\mathrm{g/mol}}{6.022\times 10^{23}\,\mathrm{mol}^{-1}} = 4.65\times 10^{-23}\,\mathrm{g} = 4.65\times 10^{-26}\,\mathrm{kg}$$

$$\boldsymbol{v}_{\boldsymbol{rms}} = \sqrt{\frac{3\boldsymbol{kT}}{\boldsymbol{m}}} = \sqrt{\frac{3\left(1.38\times 10^{-23}\,\mathrm{J/K}\right)\left(293\,\mathrm{K}\right)}{4.65\times 10^{-26}\,\mathrm{kg}}}$$

$$\boxed{\boldsymbol{v}_{\boldsymbol{rms}} = 511\,\mathrm{m/s}}$$

For Oxygen

$$\boxed{\boldsymbol{v}_{\boldsymbol{rms}} = 478\,\mathrm{m/s}}$$

Problem: 3

A mass of 135 g of an element is known to contain 30.1 x 10^{23} atoms. What is the unknown element?

$$n = \frac{N}{N_A} = \frac{30.1 \times 10^{23}}{6.022 \times 10^{23}}$$

5 moles

Since the sample has a mass of 135 g, the mass per mole is

$$\frac{135\ \text{g}}{5.00\ \text{mol}} = 27.0\ \text{g/mol}$$

Atomic mass of the substance is 27.0 u

The periodic table of the elements ➔ **Aluminum**

Problem: 16

A Goodyear blimp typically contains 5400 m^3 of helium (He) at an absolute pressure of 1.10 x 10^5 Pa. The temperature of the helium is 280 K. What is the mass (in kg) of the helium in the blimp?

$$PV = nRT$$

$$n = \frac{PV}{RT} = \frac{(1.1 \times 10^5 Pa)(5400\ m^3)}{8.31\ J/(mol.K)(280\ K)}$$

$$\boxed{2.55 \times 10^5\ moles}$$

From Periodic table →1 mole of Helium is 4.0026 g

Therefore, the mass *m* of helium in the blimp is,

$$m = (2.55 \times 10^5 mol)(4.0026 \frac{g}{mol})$$

$$= 1.0 \times 10^6 g$$

$$\boxed{1.0 \times 10^3\ kg}$$

Problem: 39 An oxygen molecule is moving near the earth's surface. Another oxygen molecule is moving in the ionosphere (the uppermost part of the earth's atmosphere) where the Kelvin temperature is 3.00 times greater. Determine the ratio of the translational rms speed in the ionosphere to that near the earth's surface.

$$\frac{1}{2} m v_{rms}^2 = \frac{3}{2} kT$$

$$(v_{rms})_{ionosphere} = \sqrt{\frac{3kT_{ionosphere}}{m}}$$

and

$$(v_{rms})_{earth} = \sqrt{\frac{3kT_{earth}}{m}}$$

Dividing the first equation by the second and using the fact that

$$T_{\text{ionosphere}} = 3T_{\text{earth's surface}}$$

$$\frac{(v_{\text{rms}})_{\text{ionosphere}}}{(v_{\text{rms}})_{\text{earth's surface}}} = \frac{\sqrt{\frac{3kT_{\text{ionosphere}}}{m}}}{\sqrt{\frac{3kT_{\text{earth's surface}}}{m}}} = \sqrt{\frac{T_{\text{ionosphere}}}{T_{\text{earth's surface}}}} = \sqrt{3} = \boxed{1.73}$$

For Practice

Ch. 14

FOC: 1, 3, 6, 7 & 8.

Problems 3, 16, & 58.