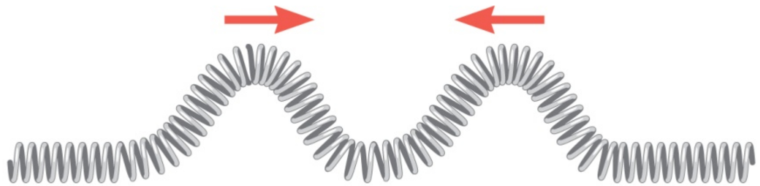
Chapter 17

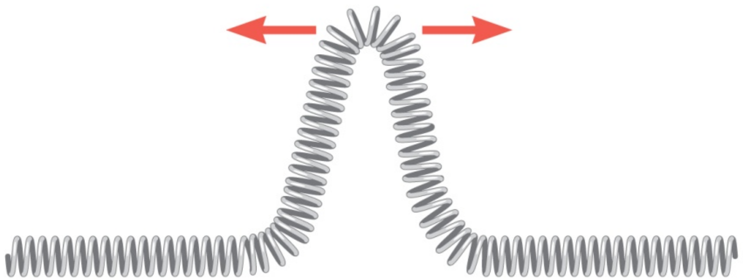
The Principle of Linear Superposition and Interference Phenomena

What happens when two or more waves are present at same point at same time?

17.1 The Principle of Linear Superposition

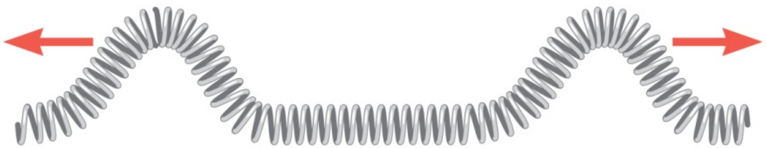


(*a*) Overlap begins

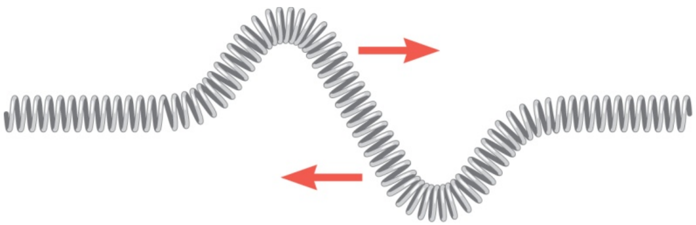


(*b*) Total overlap; the Slinky has twice the height of either pulse

When the pulses merge, the Slinky assumes a shape that is the sum of the shapes of the individual pulses.



(*c*) The receding pulses



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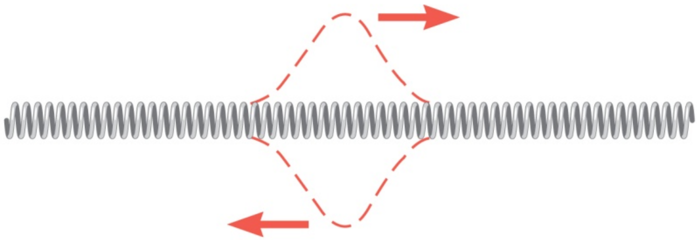
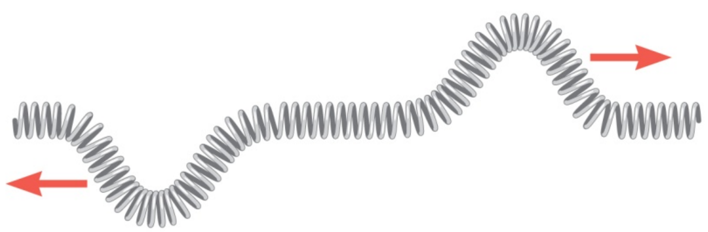


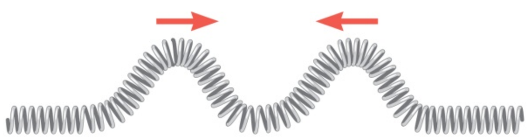
(*b*) Total overlap



(*c*) The receding pulses

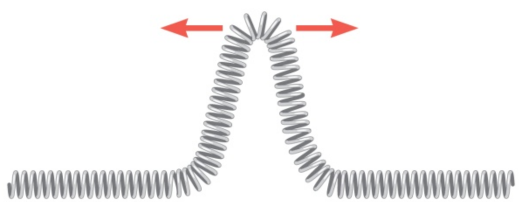
THE PRINCIPLE OF LINEAR SUPERPOSITION

When two or more waves are present simultaneously at the same place, the resultant disturbance is the sum of the disturbances from the individual waves.

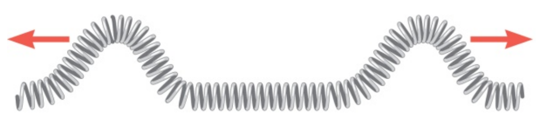


(*a*) Overlap begins

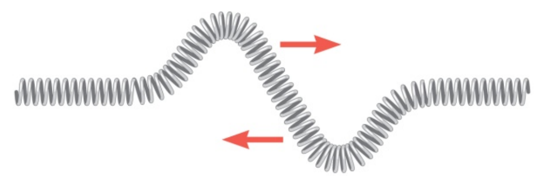
Reinforce each other



(*b*) Total overlap; the Slinky has twice the height of either pulse

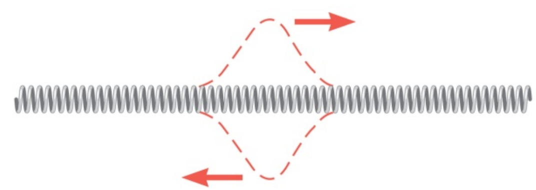


(*c*) The receding pulses



(*a*) Overlap begins

destroy each other

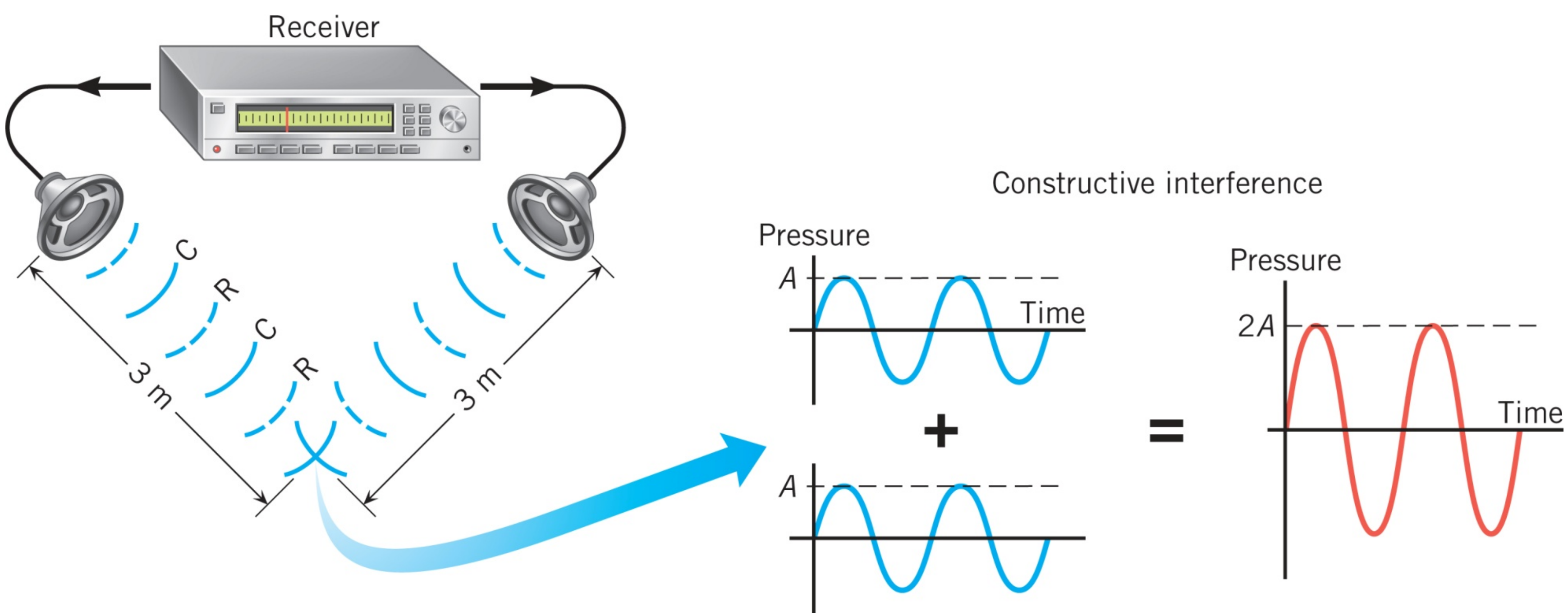


(*b*) Total overlap

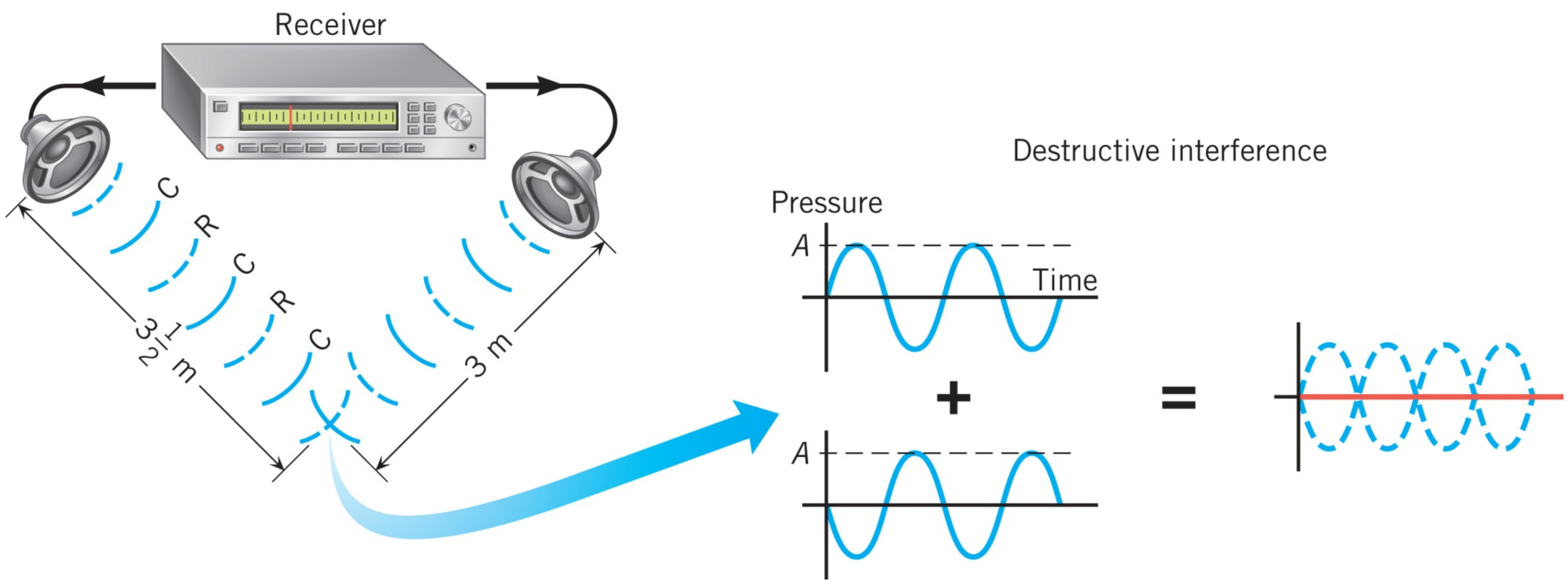
(*c*) The receding pulses

Interference of Sound Waves

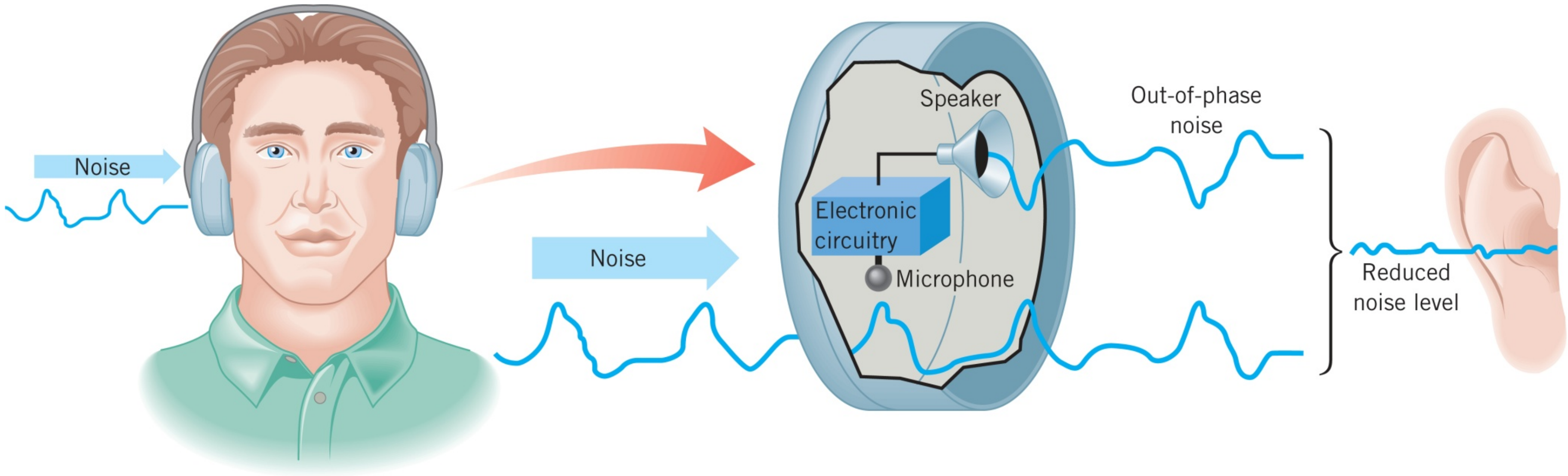
When two waves meet condensation-to-condensation and rarefaction-to-rarefaction, they are said to be ***exactly in phase*** and to exhibit ***constructive interference.***



When two waves meet condensation-to-rarefaction, they are said to be ***exactly out of phase*** and to exhibit ***destructive interference.***



A noise-cancelling headphone



If the wave patters do not shift relative to one another as time passes, the sources are said to be ***coherent***.

For two wave sources vibrating in phase, a difference in path lengths that is zero or an integer number (1, 2, 3, . .) of wavelengths leads to constructive interference; a difference in path lengths that is a half-integer number (½ , 1 ½, 2 ½, . .) of wavelengths leads to destructive interference.

For constructive interference:

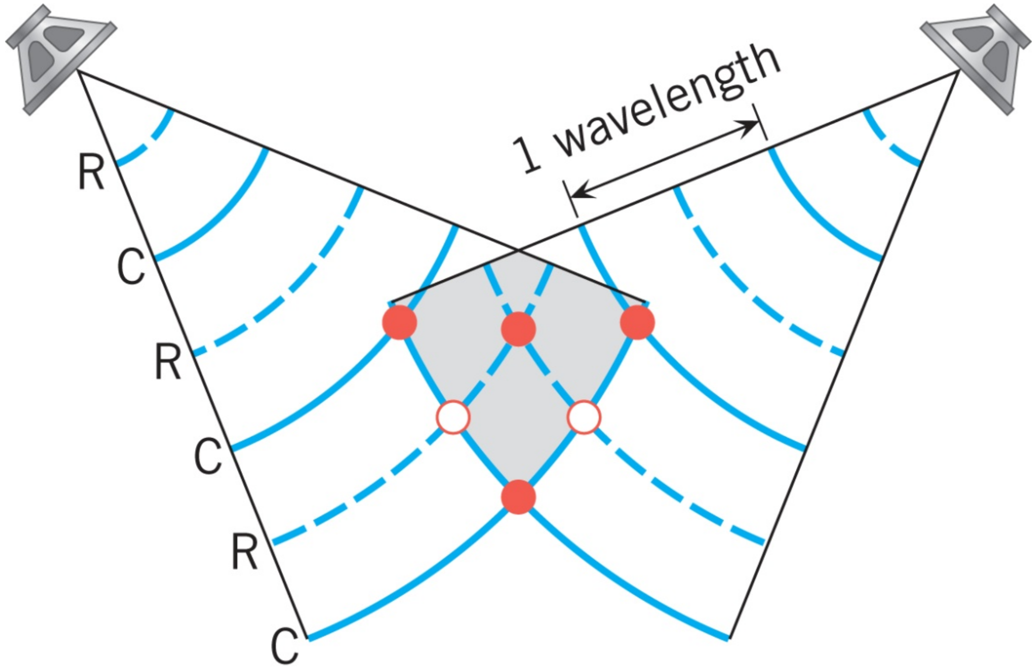
Path difference = $m\lambda$

where m is 0, 1, 2, 3, …

For destructive interference:

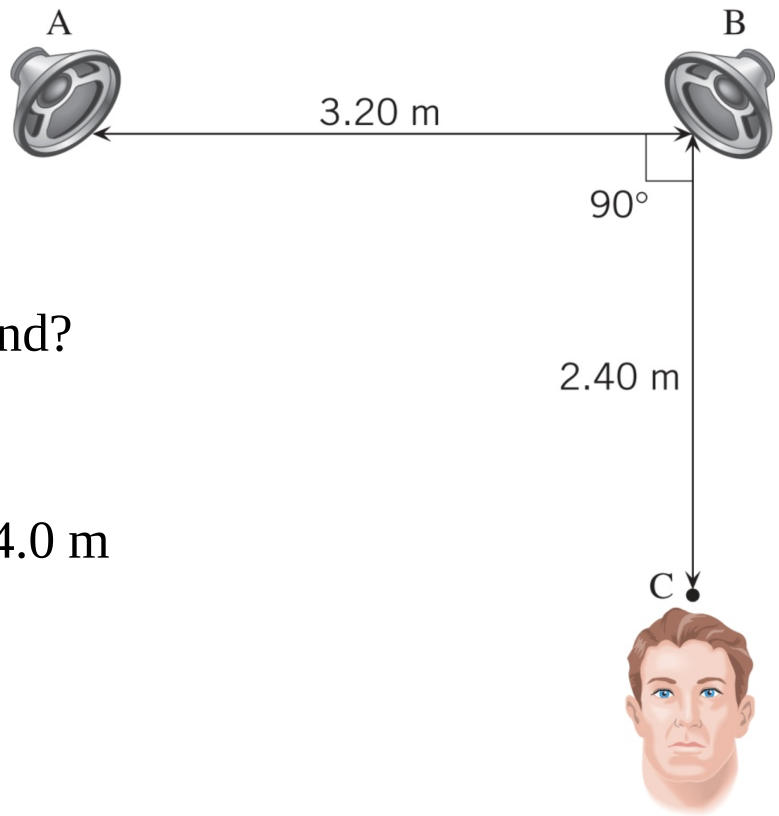
Path difference = $n\lambda/2$

where n is 1, 3, 5, 7, …



***Example*: What Does a Listener Hear?**

Two in-phase loudspeakers, A and B, are separated by 3.20 m. A listener is stationed at C, which is 2.40 m in front of speaker B. Both speakers are playing identical 214-Hz tones, and the speed of sound is 343 m/s. Does the listener hear a loud sound, or no sound?



The path length difference = AC-BC

$$\text{Distance AC} = \sqrt{(3.20\text{ m})^2 + (2.40\text{ m})^2} = 4.0\text{ m}$$

Distance BC = 2.40 m

$$\textbf{\textit{Pathdifference}} = 4.0\boldsymbol{m} - 2.40\text{m} \qquad \boxed{= 1.6\text{ m}}$$

Calculating the wavelength $\lambda = \dfrac{v}{f}$ $\quad \lambda = \dfrac{343\,\text{m/s}}{214\,\text{Hz}}$ $\quad \boxed{\lambda = 1.60\text{ m}}$

Because the path length difference is equal to an integer (1) number of wavelengths. Therefore, there is constructive interference:

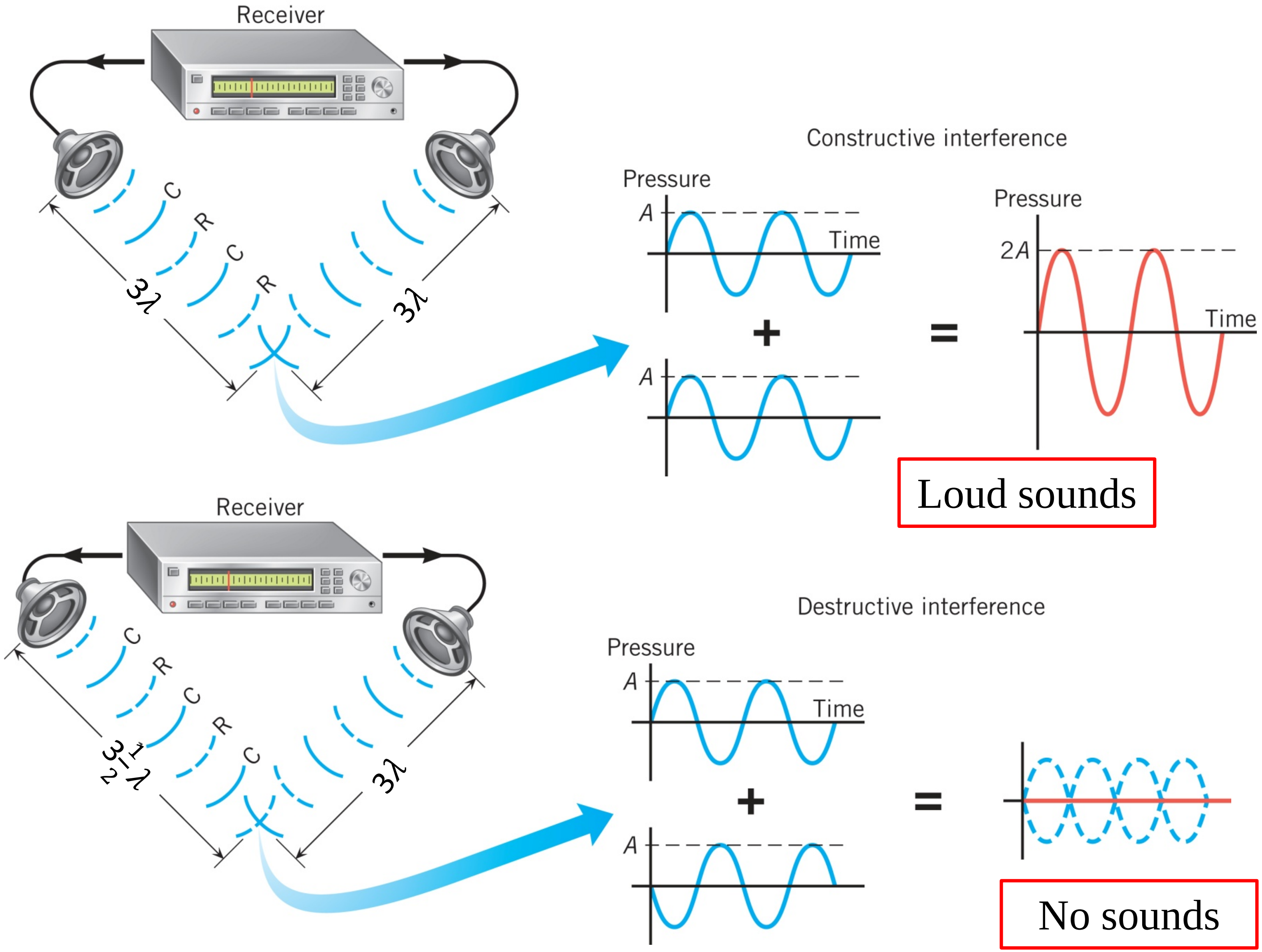
means there is a loud sound

Conceptual Example: Out-Of-Phase Speakers

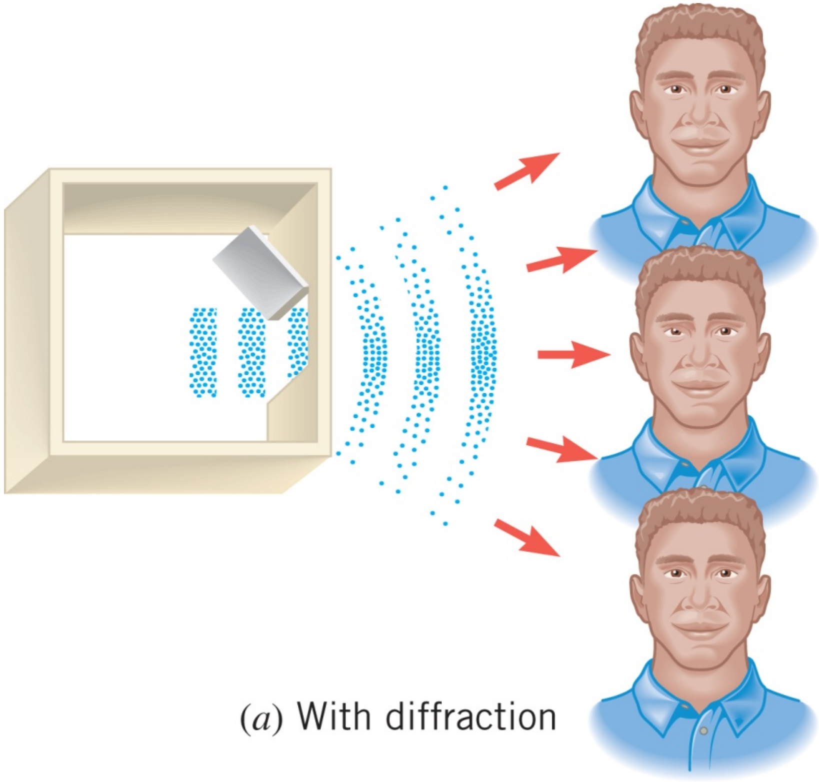
To make a speaker operate, two wires must be connected between the speaker and the amplifier. To ensure that the diaphragms of the two speakers vibrate in phase, it is necessary to make these connections in exactly the same way. If the wires for one speaker are not connected just as they are for the other, the diaphragms will vibrate out of phase. Suppose in the figures (next slide), the connections are made so that the speaker diaphragms vibrate out of phase, everything else remaining the same. In each case, what kind of interference would result in the overlap point?

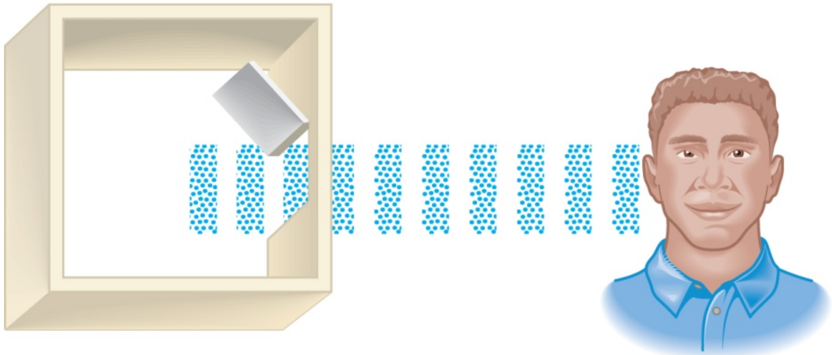
Andy Washnik

17.2 Constructive and Destructive Interference of Sound Waves



Diffraction


(*a*) With diffraction

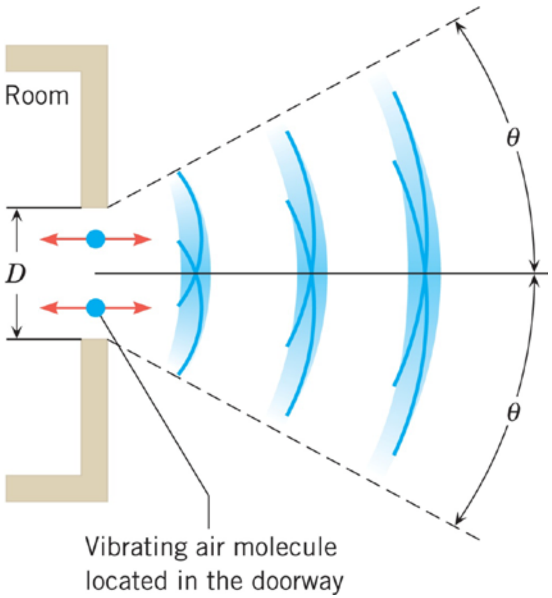

(*b*) Without diffraction

The bending of a wave around an obstacle or the edges of an opening is called ***diffraction***.

- **Diffraction**

The bending of a wave around an obstacle or the edges of an opening is called ***diffraction***.

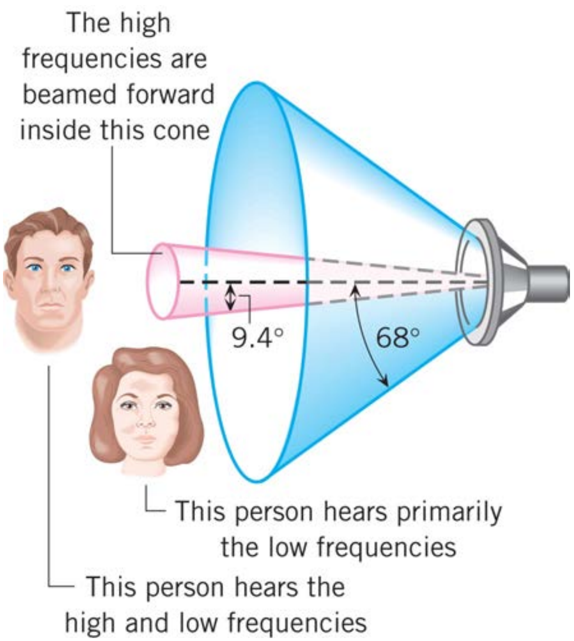
For rectangular opening of width D



single slit – first minimum

$$\boxed{\sin\theta = \frac{\lambda}{D}}$$

For circular opening of diameter D



Circular opening – first minimum

$$\boxed{\sin\theta = 1.22\frac{\lambda}{D}}$$

Problem 12:

A speaker has a diameter of 0.353 m. **(a)** Assuming that the speed of sound is 343 m/s, find the diffraction angle *θ* for a 2.0-kHz tone. **(b)** What speaker diameter *D* should be used to generate a 6.0-kHz tone whose diffraction angle is as wide as that for the 2.0-kHz tone in part **(a)**?

Circular opening $\sin\theta = 1.22\frac{\lambda}{D}$ $v = \lambda f$

(a) $f = 2.0\ kHz$ $\lambda = \frac{v}{f}$ $\lambda = \frac{343\ m/s}{2\times 10^3 Hz}$ $\boxed{= 0.17\text{ m}}$

$\theta = \sin^{-1}(1.22\frac{\lambda}{D})$ $\boxed{\theta = 36^{o}}$ For 2.0 kHz

(b) $f = 6.0\ kHz$ $\lambda = \frac{343\ m/s}{6\times 10^3 Hz}$ $\boxed{= 0.057\text{ m}}$

$\sin\theta = 1.22\frac{\lambda}{D}$ $\Rightarrow$ $D = \frac{1.22\ \lambda}{\sin\theta}$ $\boxed{D = 0.10\ m}$

Problem 14: Sound emerges through a doorway, as in Figure 17.10. The width of the doorway is 77 cm, and the speed of sound is 343 m/s. Find the diffraction angle θ when the frequency of the sound is **(a)** 5.0 kHz and **(b)** 5.0×10^2 Hz.

Rectangular opening $\sin\theta = \dfrac{\lambda}{D}$ & $\lambda = \dfrac{v}{f}$

$$\sin\theta = \frac{v}{fD}$$

(a) $$\theta = \sin^{-1}\left(\frac{v}{fD}\right)$$

$$\theta = \sin^{-1}\left[\frac{343\,\frac{m}{s}}{(5\times 10^3 Hz)(0.77m)}\right] \quad \boxed{5.1^{o}}$$

(b)

$$\theta = \sin^{-1}\left[\frac{343\,\frac{m}{s}}{(5\times 10^2 Hz)(0.77m)}\right] \quad \boxed{63^{o}}$$

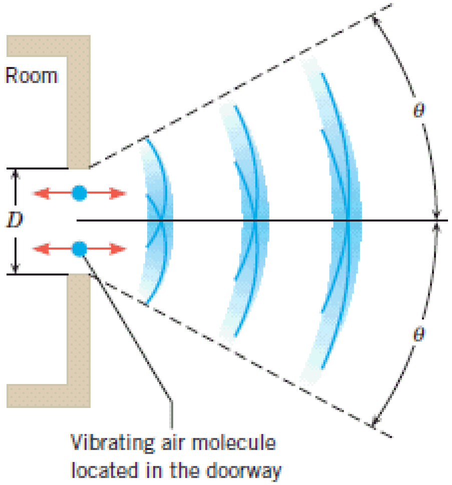
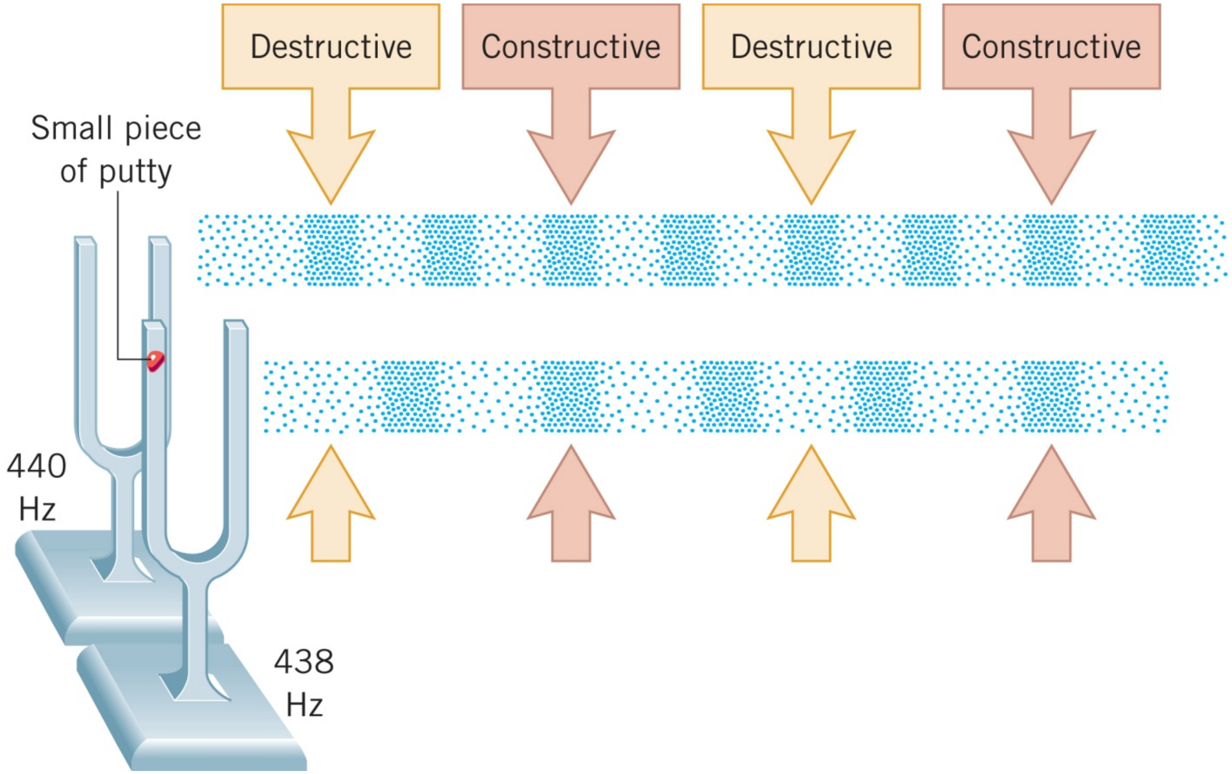
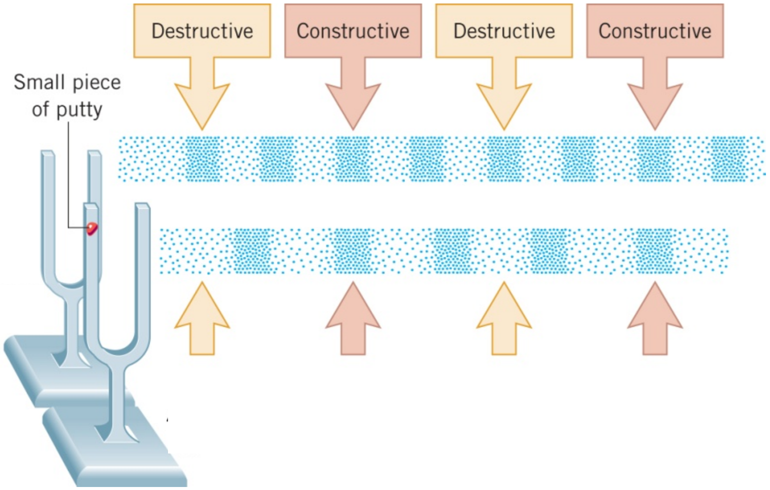


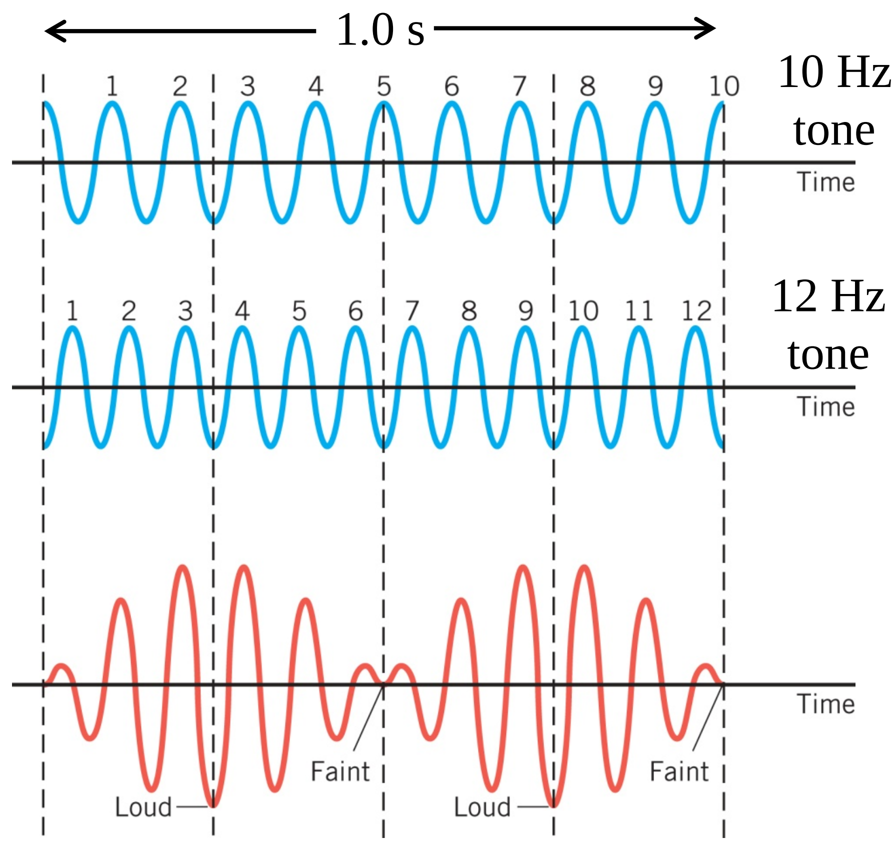
Fig. 17.10



Two overlapping waves with *slightly different frequencies* gives rise to the phenomena of beats.

17.4 Beats



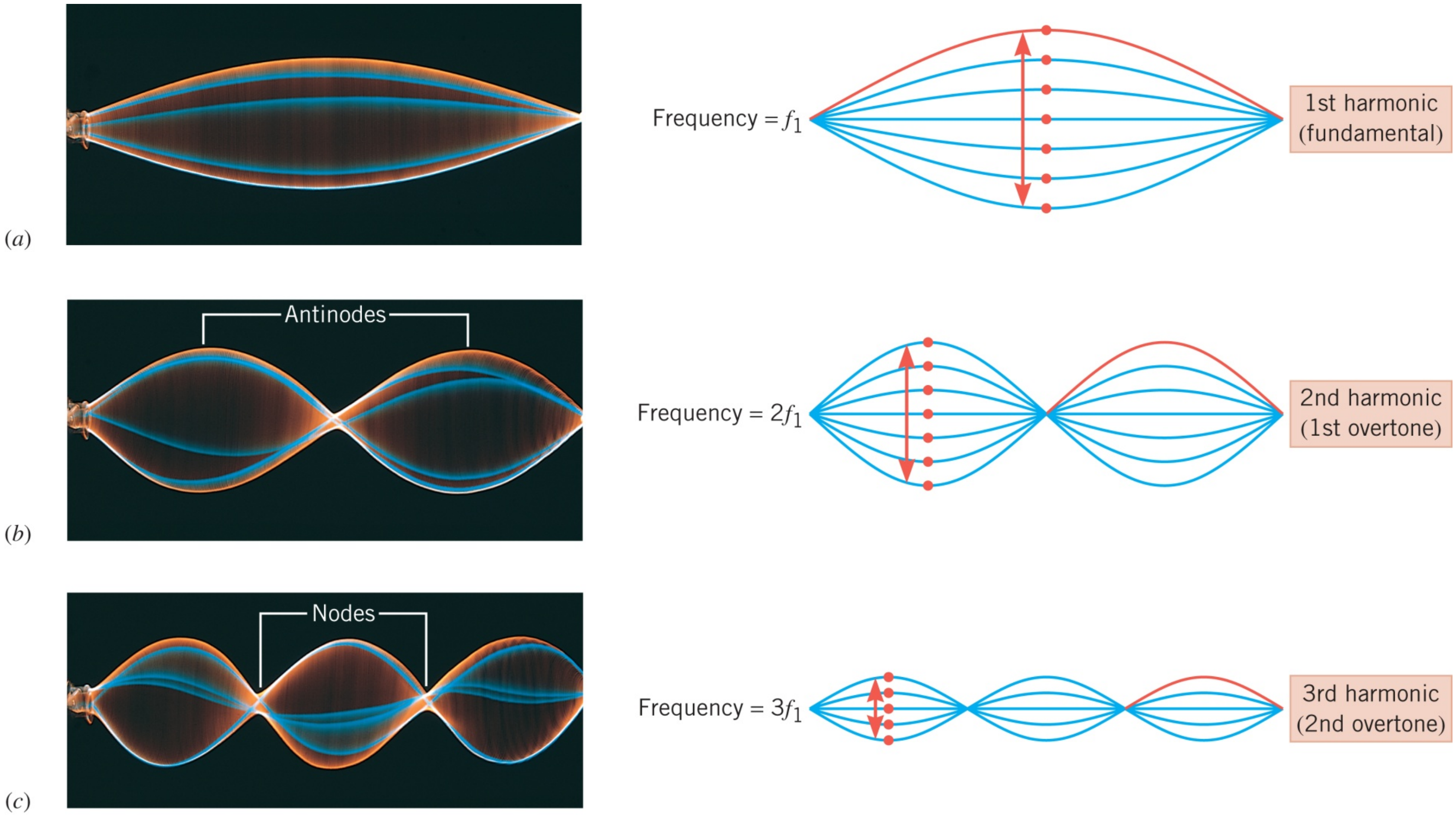


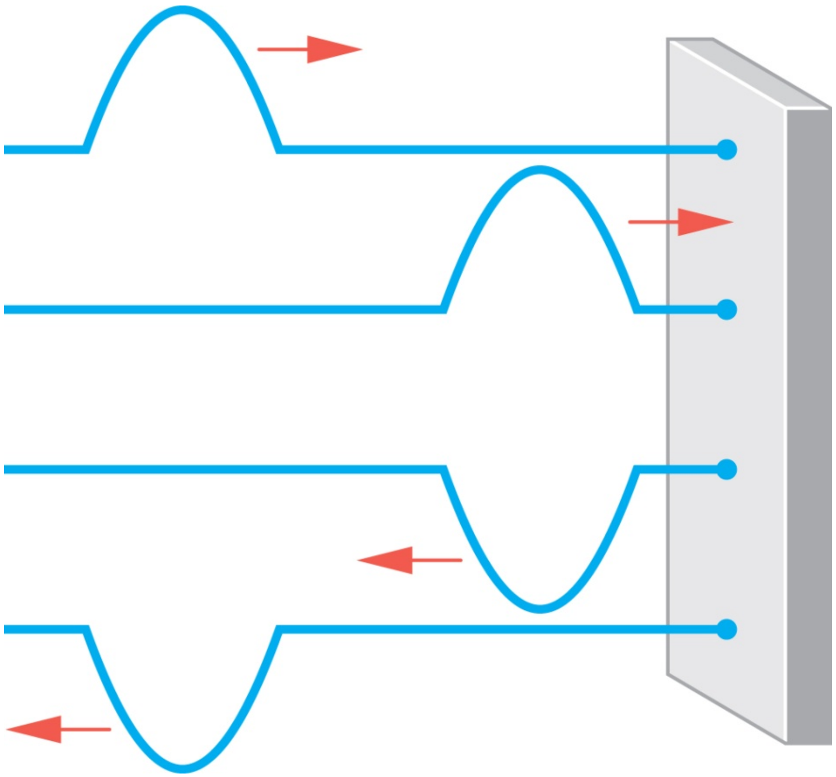
An observer hears the sound loudness rise and fall at the rate of 2 per seconds

Frequency? $\boxed{\text{2 Hz}}$

The ***beat frequency*** is the ***difference*** between the two sound frequencies.

Transverse standing wave patters





In reflecting from the wall, a forward-traveling half-cycle becomes a backward-traveling half-cycle that is inverted.

Unless the timing is right, the newly formed and reflected cycles tend to offset one another.

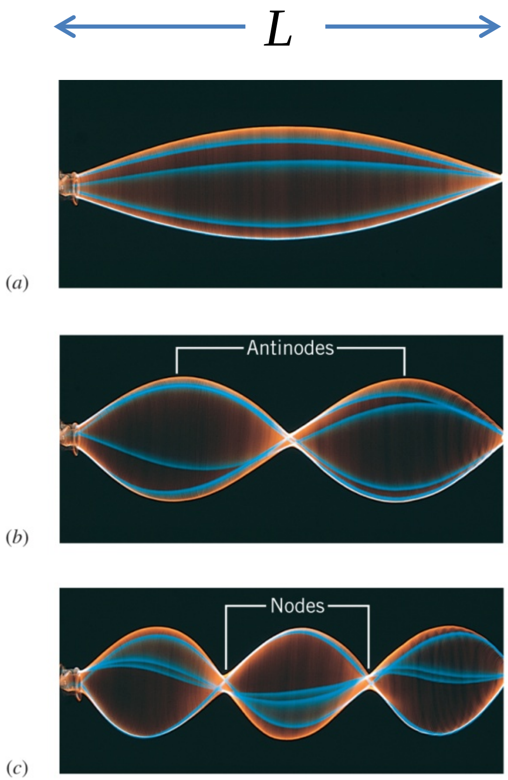
Repeated reinforcement between newly created and reflected cycles causes a large amplitude standing wave to develop.

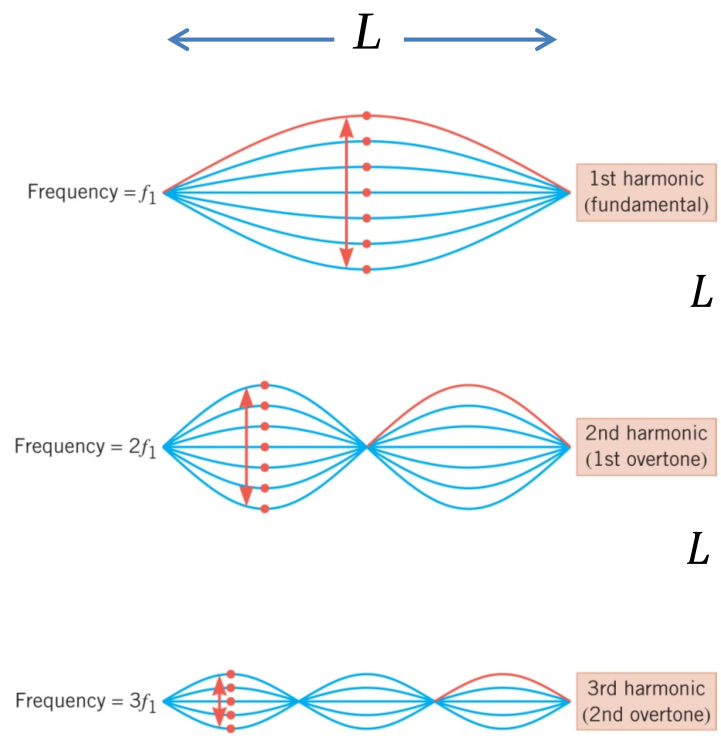
17.5 Transverse Standing Waves

$n = 1$

$n = 2$

$n = 3$





$$L = \frac{\lambda_1}{2}$$

$$L = 2\frac{\lambda_2}{2}$$

$$L = 3\frac{\lambda_3}{2}$$

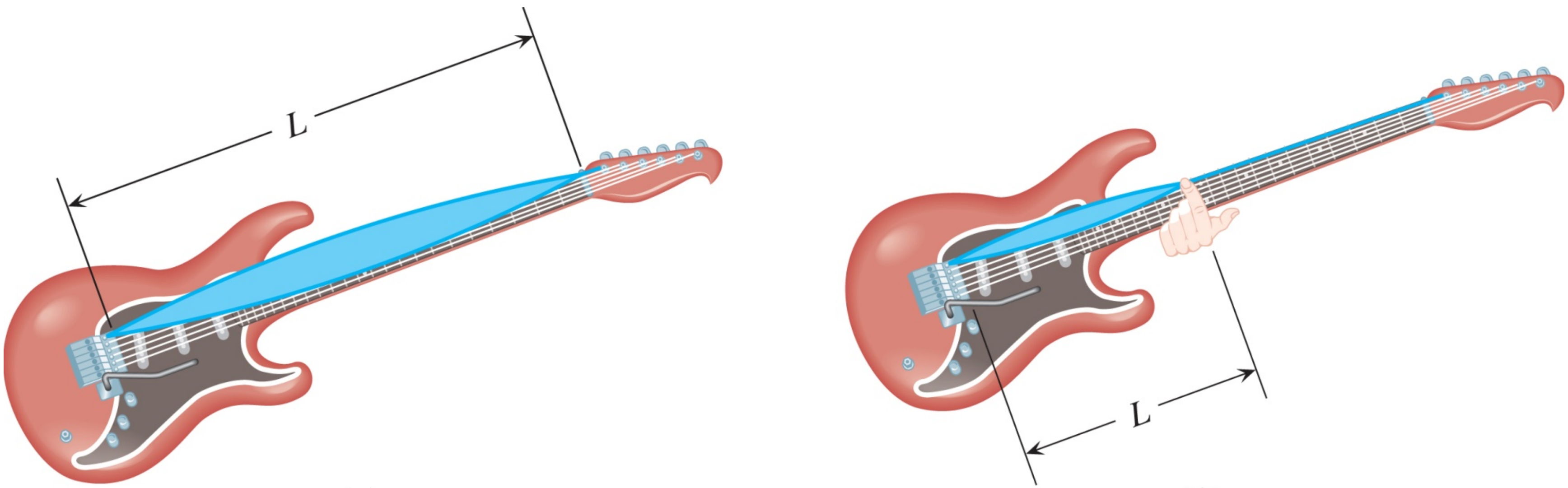
For n number of antinodes $\quad L = n\frac{\lambda_n}{2} \Rightarrow \lambda_n = \frac{2L}{n}$

Since $v = \lambda f$ OR $f = \frac{v}{\lambda}$

String fixed at both ends

$$f_n = n\left(\frac{v}{2L}\right) \qquad n = 1, 2, 3, 4, \ldots$$

$$f_n = nf_1$$



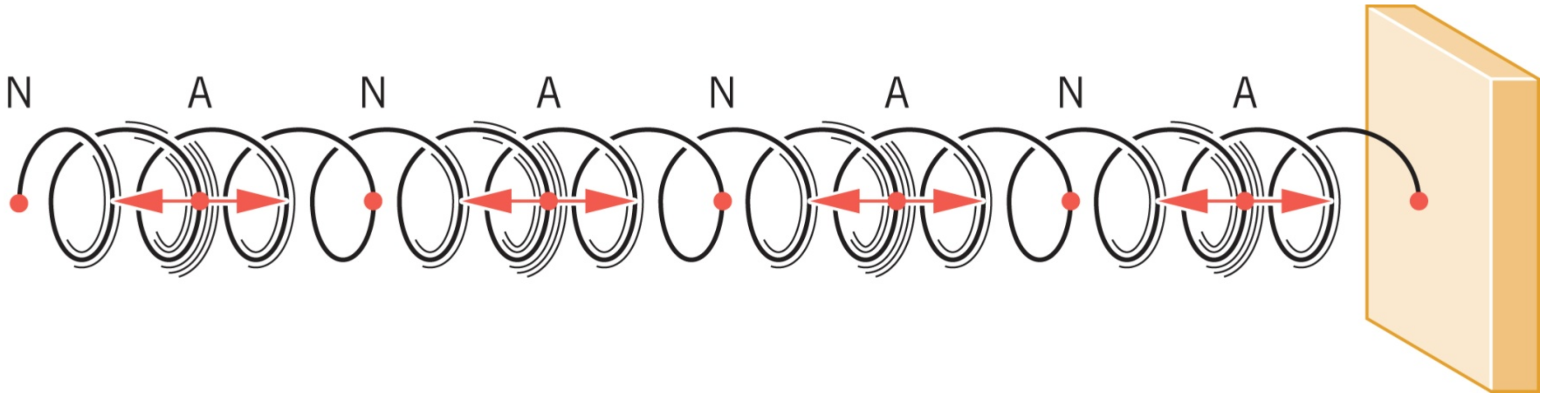
(a) (b)

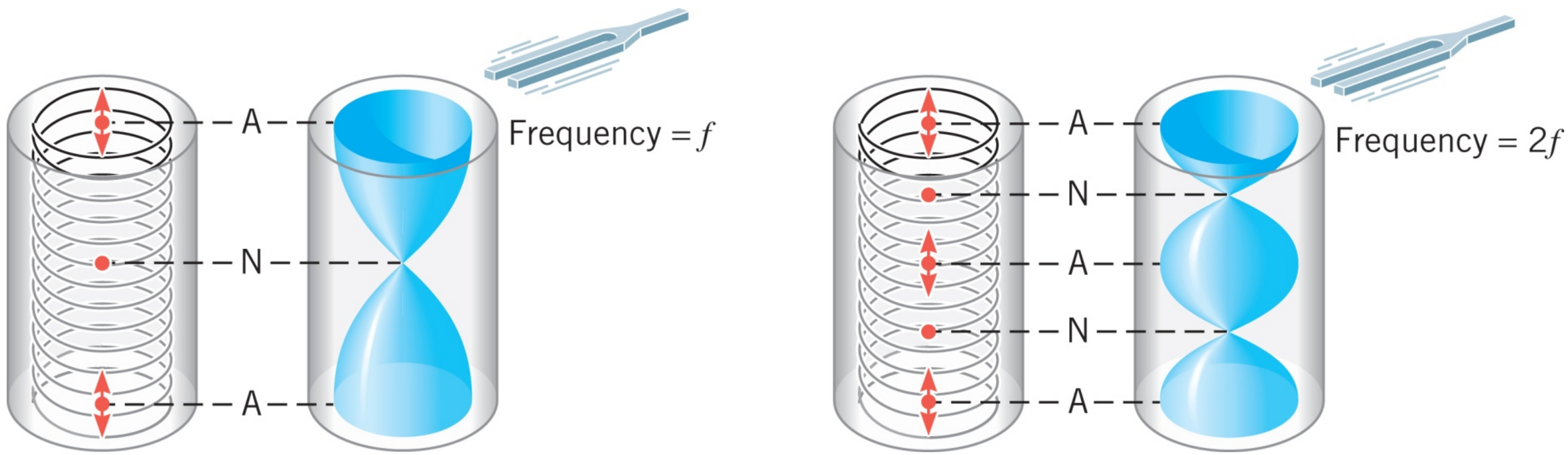
$$f_n = n\left(\frac{v}{2L}\right) \qquad n = 1, 2, 3, 4, \ldots$$

$$f_1 = fundamental\ frequency\ OR\ First\ harmonics = \frac{v}{2L}$$

$$f_n = n f_1$$

A longitudinal standing wave pattern on a slinky.



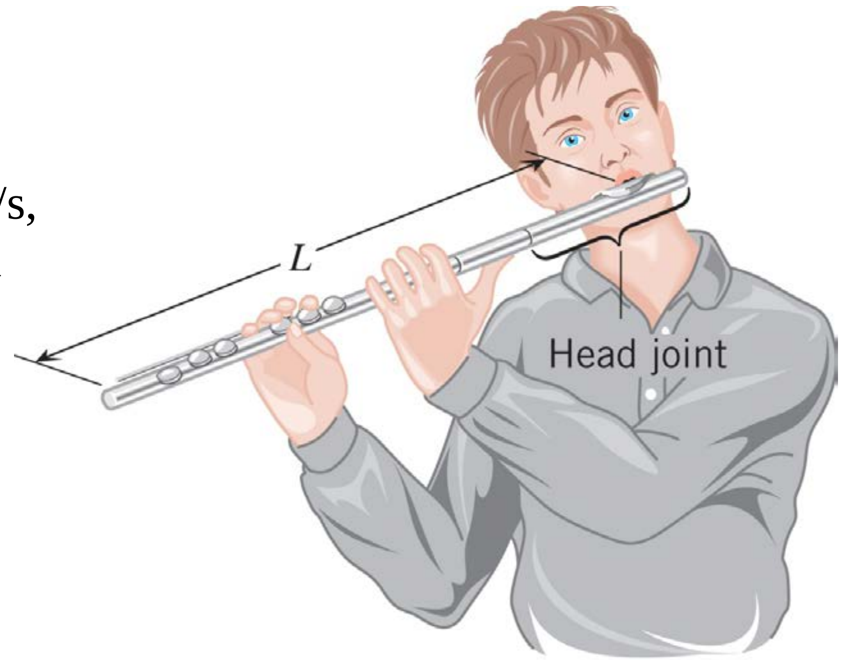


Tube open at both ends

$$f_n = n\left(\frac{v}{2L}\right) \qquad n = 1, 2, 3, 4, \ldots$$

Fixed at both ends or free (open tube) at both ends

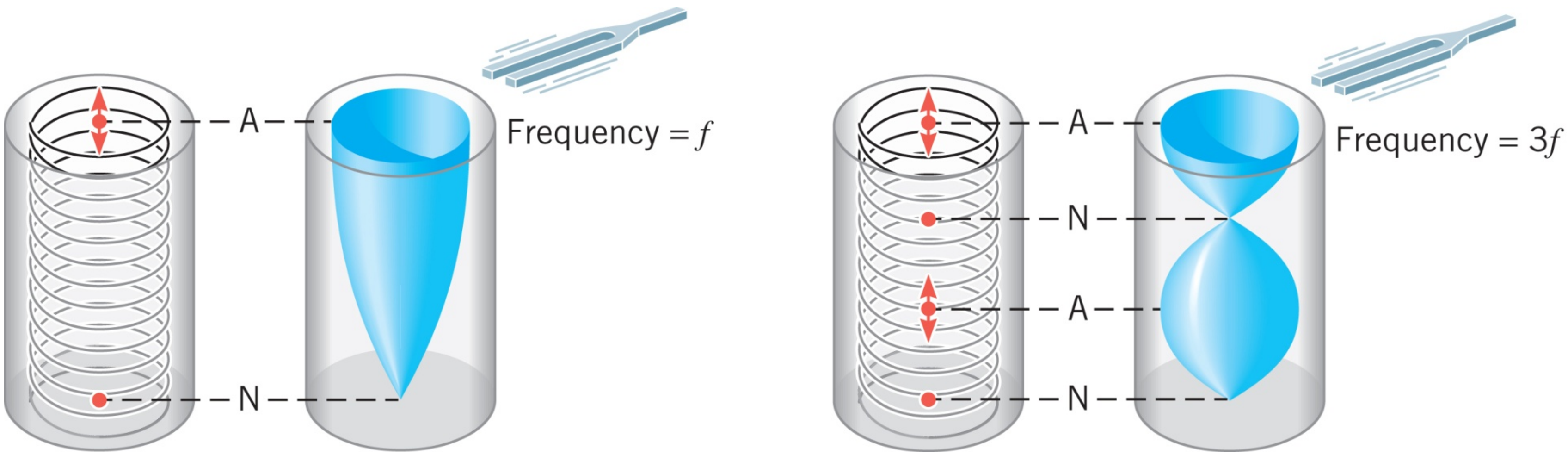
Example: Playing a Flute
When all the holes are closed on one type of flute, the lowest note it can sound is middle C (261.6 Hz). If the speed of sound is 343 m/s, and the flute is assumed to be a cylinder open at both ends, determine the distance L.



$$f_n = n\left(\frac{v}{2L}\right) \qquad n = 1, 2, 3, 4, \ldots$$

$$L = \frac{nv}{2f_n} = \frac{1(343\,\mathrm{m/s})}{2(261.6\,\mathrm{Hz})}$$

$$\boxed{L = 0.656\,\mathrm{m}}$$



Tube open at one end

$$f_n = n\left(\frac{v}{4L}\right) \qquad n = 1, 3, 5, \ldots$$

Or string fixed at one end and free at other end

Problem: 45

The fundamental frequencies of two air columns are same. Column A is open at both ends, while column B is open at only one end. The length of column A is 0.70 m. What is the length of column B?

Since the fundamental frequencies of the two air columns are the same

$$f_1^{\mathrm{A}} = f_1^{\mathrm{B}}$$

$$f_1^{\mathrm{A}} = (1)\left(\frac{v}{2L_{\mathrm{A}}}\right) \quad \text{and} \quad f_1^{\mathrm{B}} = (1)\left(\frac{v}{4L_{\mathrm{B}}}\right)$$

Therefore

$$\frac{v}{2L_A} = \frac{v}{4L_B}$$

$$L_B = \frac{1}{2}L_A \qquad L_B = \frac{1}{2}(0.70\ m) \qquad \boxed{= 0.35\ \mathrm{m}}$$

Problem: 53

A string is fixed from both ends and is vibrating at 130 Hz, which is it's 3^{rd} harmonic frequency. The linear density of the string is 5.6 x 10^{-3} kg/m, and it is under a tension of 3.3 N. Determine the length of the string.

$$f_3 = 3\left(\frac{v}{2L}\right) \quad \text{OR} \quad L = \frac{3v}{2f_3} \quad \text{and} \quad v = \sqrt{\frac{F}{m/L}}$$

$$L = \frac{3v}{2\,f_3} = \frac{3}{2\,f_3}\sqrt{\frac{F}{m/L}}$$

Although L appears on both sides of Equation (2), no further algebra is required. This is because L appears in the ratio m/L on the right side. This ratio is the linear density of the string, which has a known value of 5.6×10−3 kg/m. Therefore, the length of the string is

$$L = \frac{3}{2(130\ Hz)}\sqrt{\frac{3.3\ N}{5.6 \times 10^{-3} kg/m}} \quad \boxed{= 0.28\text{ m}}$$

Problem: 15

The entrance to a large lecture room consists of two side-by-side doors, one hinged on the left and the other hinged on the right. Each door is 0.700 m wide. Sound of the frequency 607 Hz is coming through the entrance from within the room. The speed of sound is 343 m/s. What is the diffraction angle of the sound after it passes through the doorway when

(a) One door is open and

(b) Both doors are open.

(a) $\sin\theta = \frac{\lambda}{D}$ and $v = \lambda f$ $\sin\theta = \frac{v}{fD}$

$$\sin\theta = \frac{343\ m/s}{(607\ Hz)(0.70m)}$$

$$\boxed{\theta = 53.8°}$$

(b) When both doors are open, D = 2 × 0.700 m and the diffraction angle is

$$\sin\theta = \frac{343\ m/s}{(607\ Hz)(2 \times 0.70m)}$$

$$\boxed{\theta = 23.8°}$$

For Recitation

Ch. 17

FOC: 2, 5, 11 &12.

Problems: 7, 14, 41 & 43.

Exam#4 on Friday

Ch. 12 to Ch. 17