

Ch. 12 TEMPERATURE and HEAT

Phys 131 Recitation

For Recitations

Ch. 12

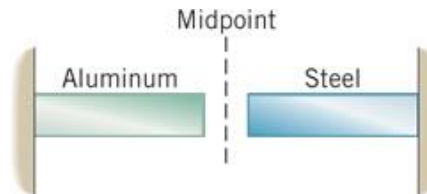
FOC Q: 2, 4, 7, 9 & 10.

Problems: 12, 15, 19, 28, 45 & 68

FOC 2

Section 12.4 Linear Thermal Expansion

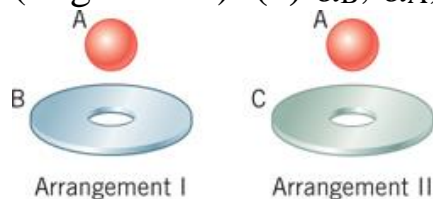
2. The drawing shows two thin rods, one made from aluminum [$\alpha = 23 \times 10^{-6} (\text{C}^\circ)^{-1}$] and the other from steel [$\alpha = 12 \times 10^{-6} (\text{C}^\circ)^{-1}$]. Each rod has the same length and the same initial temperature and is attached at one end to an immovable wall, as shown. The temperatures of the rods are increased, both by the same amount, until the gap between the rods is closed. Where do the rods meet when the gap is closed? **(a)** The rods meet exactly at the midpoint. **(b)** The rods meet to the right of the midpoint. **(c)** The rods meet to the left of the midpoint.



2. (b) According to Equation 12.2, the change in length ΔL of each rod is given by $\Delta L = \alpha L_0 \Delta T$, where α is the coefficient of linear expansion, L_0 is the initial length, and ΔT is the change in temperature. Since the initial length and the change in temperature are the same for each rod, the rod with the larger coefficient of linear expansion has the greater increase in length as the temperature rises. Thus, the aluminum rod lengthens more than the steel rod, so the rods will meet to the right of the midpoint.

FOC 4

4. A ball is slightly too large to fit through a hole in a flat plate. The drawing shows two arrangements of this situation. In Arrangement I the ball is made from metal A and the plate from metal B. When both the ball and the plate are cooled by the same number of Celsius degrees, the ball passes through the hole. In Arrangement II the ball is also made from metal A, but the plate is made from metal C. Here, the ball passes through the hole when both the ball and the plate are heated by the same number of Celsius degrees. Rank the coefficients of linear thermal expansion of metals A, B, and C in descending order (largest first): **(a)** $\alpha_B, \alpha_A, \alpha_C$ **(b)** $\alpha_B, \alpha_C, \alpha_A$ **(c)** $\alpha_C, \alpha_B, \alpha_A$ **(d)** $\alpha_C, \alpha_A, \alpha_B$ **(e)** $\alpha_A, \alpha_B, \alpha_C$



4. (d) In Arrangement I cooling allows the ball to pass through the hole. Therefore, the ball must shrink more than the hole, and the coefficient of linear thermal expansion of metal A must be greater than that of metal B. In Arrangement II heating allows the ball to pass through the hole. Therefore, the coefficient of linear thermal expansion of metal C must be greater than that of metal A.

FOC 7

7. A container can be made from steel [$\beta = 36 \times 10^{-6} (\text{C}^\circ)^{-1}$] or lead [$\beta = 87 \times 10^{-6} (\text{C}^\circ)^{-1}$]. A liquid is poured into the container, filling it to the brim. The liquid is either water [$\beta = 207 \times 10^{-6} (\text{C}^\circ)^{-1}$] or ethyl alcohol [$\beta = 1120 \times 10^{-6} (\text{C}^\circ)^{-1}$]. When the full container is heated, some liquid spills out. To keep the overflow to a minimum, from what material should the container be made and what should the liquid be? **(a)** Lead, water **(b)** Steel, water **(c)** Lead, ethyl alcohol **(d)** Steel, ethyl alcohol

7. (a) To keep the overflow to a minimum, the container should be made from a material that has the greatest coefficient of volume thermal expansion and filled with a liquid that has the smallest coefficient of volume thermal expansion. That way, when the full container is heated, the cavity holding the liquid will expand more and the liquid will expand less, both effects leading to a reduced amount of overflow.

FOC 9

Section 12.7 Heat and Temperature Change: Specific Heat Capacity

9. Which of the following cases (if any) requires the greatest amount of heat? In each case the material is the same. (a) 1.5 kg of the material is to be heated by 7.0 C°. (b) 3.0 kg of the material is to be heated by 3.5 C°. (c) 0.50 kg of the material is to be heated by 21 C°. (d) 0.75 kg of the material is to be heated by 14 C°. (e) The amount of heat required is the same in each of the four previous cases.

9. (e) The heat Q required to raise the temperature of a mass m of material by an amount ΔT is given by Equation 12.4 as $Q = cm\Delta T$, where c is the specific heat capacity of the material. Since the material is the same in all cases, the specific heat capacity is the same. What matters is the product of m and ΔT . Since this product is the same in all cases, the amount of heat needed is also the same.

FOC 10

10. The following three hot samples have the same temperature. The same amount of heat is removed from each sample. Which one experiences the smallest drop in temperature, and which one experiences the largest drop?

Sample A. 4.0 kg of water [$c = 4186 \text{ J}/(\text{kg} \cdot \text{C}^\circ)$]

Sample B. 2.0 kg of oil [$c = 2700 \text{ J}/(\text{kg} \cdot \text{C}^\circ)$]

Sample C. 9.0 kg of dirt [$c = 1050 \text{ J}/(\text{kg} \cdot \text{C}^\circ)$]

(a) C smallest and A largest **(b)** B smallest and C largest **(c)** A smallest and B largest **(d)** C smallest and B largest **(e)** B smallest and A largest

10. (c) The samples cool as heat is removed from each one. However, the temperature change that results as heat is removed is different. The heat Q that must be removed to lower the temperature of a mass m of material by an amount ΔT is given by Equation 12.4 as $Q = cm\Delta T$, where c is the specific heat capacity of the material. Solving for ΔT gives $\Delta T = Q/(cm)$. For a given amount of heat removed, the fall in temperature is inversely proportional to the product cm . The sample (sample A) with the largest value of cm will experience the smallest drop in temperature. The sample (sample B) with the smallest value of cm will experience the largest drop in temperature.

Pr. 12

12. The Eiffel Tower is a steel structure whose height increases by 19.4 cm when the temperature changes from -9 to $+41$ °C. What is the approximate height (in meters) at the lower temperature?

2. **REASONING** The height L_0 of the Eiffel Tower at the lower temperature can be determined from $L_0 = \Delta L / (\alpha \Delta T)$ (Equation 12.2), where ΔL is the increase in the height, α is the coefficient of linear expansion for steel, and ΔT is the change in temperature. The coefficient of linear expansion for steel can be found in Table 12.1.

SOLUTION The height of the Eiffel Tower at the lower temperature is

$$L_0 = \frac{\Delta L}{\alpha \Delta T} = \frac{19.4 \times 10^{-2} \text{ m}}{\left[1.2 \times 10^{-5} (\text{C}^\circ)^{-1}\right] \left[41 \text{ }^\circ\text{C} - (-9 \text{ }^\circ\text{C})\right]} = \boxed{3.2 \times 10^2 \text{ m}}$$


Pr. 15

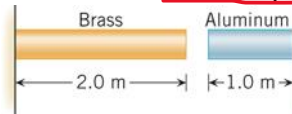
15. ssm When the temperature of a coin is raised by 75 C° , the coin's diameter increases by $2.3 \times 10^{-5}\text{ m}$. If the original diameter of the coin is $1.8 \times 10^{-2}\text{ m}$, find the coefficient of linear expansion.

15. **SSM** *REASONING AND SOLUTION* The change in the coin's diameter is $\Delta d = \alpha d_0 \Delta T$, according to Equation 12.2. Solving for α gives

$$\alpha = \frac{\Delta d}{d_0 \Delta T} = \frac{2.3 \times 10^{-5}\text{ m}}{(1.8 \times 10^{-2}\text{ m})(75\text{ C}^\circ)} = \boxed{1.7 \times 10^{-5}\text{ (C}^\circ)^{-1}}$$

Pr. 19

*19.  The brass bar and the aluminum bar in the drawing are each attached to an immovable wall. At 28 °C the air gap between the rods is 1.3×10^{-3} m. At what temperature will the gap be closed?



19. **REASONING AND SOLUTION** $\Delta L = \alpha L_0 \Delta T$ gives for the expansion of the aluminum

$$\Delta L_A = \alpha_A L_A \Delta T$$

and for the expansion of the brass

$$\Delta L_B = \alpha_B L_B \Delta T$$

The air gap will be closed when $\Delta L_A + \Delta L_B = 1.3 \times 10^{-3}$ m. Thus, taking the coefficients of thermal expansion for aluminum and brass from Table 12.1, adding Equations (1) and (2), and solving for ΔT , we find that

$$\Delta T = \frac{\Delta L_A + \Delta L_B}{\alpha_A L_A + \alpha_B L_B} = \frac{1.3 \times 10^{-3} \text{ m}}{[23 \times 10^{-6} (\text{C}^\circ)^{-1}](1.0 \text{ m}) + [19 \times 10^{-6} (\text{C}^\circ)^{-1}](2.0 \text{ m})} = 21 \text{ C}^\circ$$

The desired temperature is then

$$T = 28 \text{ }^\circ\text{C} + 21 \text{ C}^\circ = \boxed{49 \text{ }^\circ\text{C}}$$

Pr. 28

Section 12.5 Volume Thermal Expansion

28. A flask is filled with 1.500 L (L = liter) of a liquid at 97.1 °C. When the liquid is cooled to 15.0 °C, its volume is only 1.383 L, however. Neglect the contraction of the flask and use Table 12.1 to identify the liquid.

28. **REASONING** We can identify the liquid by computing the coefficient of volume expansion β and then comparing the result with the values of β given in Table 12.1. The relation $\Delta V = \beta V_0 \Delta T$ (Equation 12.3) can be used to calculate β .

SOLUTION Solving Equation 12.3 for β , we have that $\beta = \Delta V / (V_0 \Delta T)$. The change in the volume of the liquid is $\Delta V = 1.500 \text{ L} - 1.383 \text{ L} = 0.117 \text{ L}$. Therefore, the coefficient of volume expansion for the unknown liquid is

$$\beta = \frac{\Delta V}{V_0 \Delta T} = \frac{0.117 \text{ L}}{(1.500 \text{ L})(97.1 \text{ }^\circ\text{C} - 15.0 \text{ }^\circ\text{C})} = 9.50 \times 10^{-4} \text{ (}^\circ\text{C)}^{-1} = 950 \times 10^{-6} \text{ (}^\circ\text{C)}^{-1}$$

A comparison with the values of β in Table 12.1 indicates that the liquid is gasoline

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Pr. 45

45. An ice chest at a beach party contains 12 cans of soda at $5.0\text{ }^{\circ}\text{C}$. Each can of soda has a mass of 0.35 kg and a specific heat capacity of $3800\text{ J}/(\text{kg} \cdot ^{\circ}\text{C})$. Someone adds a 6.5-kg watermelon at $27\text{ }^{\circ}\text{C}$ to the chest. The specific heat capacity of watermelon is nearly the same as that of water. Ignore the specific heat capacity of the chest and determine the final temperature T of the soda and watermelon.

45. REASONING We assume that no heat is lost through the chest to the outside. Then, energy conservation dictates that the heat gained by the soda is equal to the heat lost by the watermelon in reaching the final temperature T_f . Each quantity of heat is given by Equation 12.4, $Q = cm\Delta T$, where we write the change in temperature ΔT as the higher temperature minus the lower temperature.

SOLUTION Starting with the statement of energy conservation, we have

Heat gained by soda = Heat lost by watermelon

$$(cm\Delta T)_{\text{soda}} = (cm\Delta T)_{\text{watermelon}}$$

Since the watermelon is being treated like water, we take the specific heat capacity of water from Table 12.2. Thus, the above equation becomes

$$[3800\text{ J}/(\text{kg} \cdot ^{\circ}\text{C})](12 \times 0.35\text{ kg})(T_f - 5.0\text{ }^{\circ}\text{C}) = [4186\text{ J}/(\text{kg} \cdot ^{\circ}\text{C})](6.5\text{ kg})(27\text{ }^{\circ}\text{C} - T_f)$$


Suppressing units for convenience and algebraically simplifying, we have

$$1.6 \times 10^4 T_f - 8.0 \times 10^4 = 7.3 \times 10^5 - 2.7 \times 10^4 T_f$$

Solving for T_f , we obtain

$$T_f = \frac{8.1 \times 10^5}{4.3 \times 10^4} = \boxed{19\text{ }^{\circ}\text{C}}$$

Pr. 68

*68.  Water at 23.0 °C is sprayed onto 0.180 kg of molten gold at 1063 °C (its melting point). The water boils away, forming steam at 100.0 °C and leaving solid gold at 1063 °C. What is the minimum mass of water that must be used?

Change of phase \rightarrow no ΔT

68. **REASONING** When the minimum mass m_w of water is used to solidify the molten gold, the heat Q_g lost by the gold is exactly the amount needed to raise the temperature of the water to the boiling point and then convert the water into steam. If the mass of the water is smaller than m_w , all of the water will be converted to steam before the gold has entirely solidified. The water first gains an amount of heat $Q_1 = c_w m_w \Delta T$ (Equation 12.4) when it is raised to the boiling point (100.0 °C), where $c_w = 4186 \text{ J}/(\text{kg} \cdot \text{C}^\circ)$ is the specific heat capacity of water (see Table 12.2) and ΔT is the change in its temperature. When the boiling water is converted to steam, it absorbs a further amount of heat $Q_2 = m_w L_w$ (Equation 12.5), where $L_w = 22.6 \times 10^5 \text{ J/kg}$ is the latent heat of vaporization of water (see Table 12.3). The total amount of heat given off by the molten gold as it solidifies is found from $Q_g = m_g L_g$ (Equation 12.5), where m_g is the mass of the gold and $L_g = 6.28 \times 10^4 \text{ J/kg}$ is the latent heat of fusion of gold (see Table 12.3). The total amount of heat Q_w gained by the water is equal to the heat lost by the gold, so we have that

$$Q_w = Q_1 + Q_2 = Q_g$$

Pr. 68 Solution

SOLUTION Substituting $Q_1 = c_w m_w \Delta T$ (Equation 12.4), $Q_2 = m_w L_w$ (Equation 12.5), and $Q_g = m_g L_g$ (Equation 12.5) into Equation (1), we obtain

$$\underbrace{c_w m_w \Delta T + m_w L_w}_{\text{Heat gained by water}} = \underbrace{m_g L_g}_{\text{Heat lost by gold}} \quad (2)$$

Solving Equation (2) for m_w yields

$$m_w (c_w \Delta T + L_w) = m_g L_g \quad \text{or} \quad m_w = \frac{m_g L_g}{c_w \Delta T + L_w}$$

Therefore,

$$m_w = \frac{(0.180 \text{ kg})(6.28 \times 10^4 \text{ J/kg})}{[4186 \text{ J/(kg} \cdot \text{C}^\circ)](100.0 \text{ }^\circ\text{C} - 23.0 \text{ }^\circ\text{C}) + 22.6 \times 10^5 \text{ J/kg}} = \boxed{4.38 \times 10^{-3} \text{ kg}}$$

Ch. 13 THE TRANSFER OF HEAT

Phys 131 Recitation

Natalia Dushkina

For Practice

Ch. 13

FOC: 1, 2, 8 & 11.

Problems: 10, 12, 20, 25 & 42

FOC 1

Section 13.2 Conduction

1. The heat conducted through a bar depends on which of the following?

A. The coefficient of linear expansion

B. The thermal conductivity

C. The specific heat capacity

D. The length of the bar

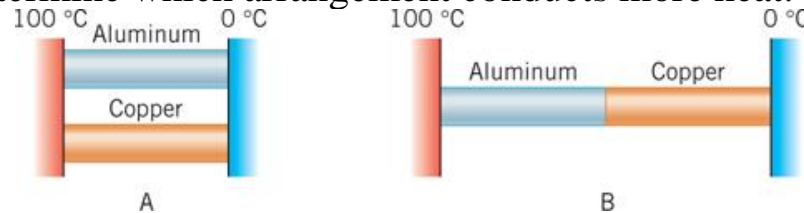
E. The cross-sectional area of the bar

(a) A, B, and D (b) A, C, and D (c) B, C, D, and E (d) B, D, and E (e) C, D, and E

1. (d) The heat conducted during a time t through a bar is given by $Q = \frac{(kA\Delta T)t}{L}$, where k is the thermal conductivity, and A and L are the cross-sectional area and length of the bar.

FOC 2

2. Two bars are conducting heat from a region of higher temperature to a region of lower temperature. The bars have identical lengths and cross-sectional areas, but are made from different materials. In the drawing they are placed “in parallel” between the two temperature regions in arrangement A, whereas they are placed end to end in arrangement B. In which arrangement is the heat that is conducted the greatest? (a) The heat conducted is the same in both arrangements. (b) Arrangement A (c) Arrangement B (d) It is not possible to determine which arrangement conducts more heat.



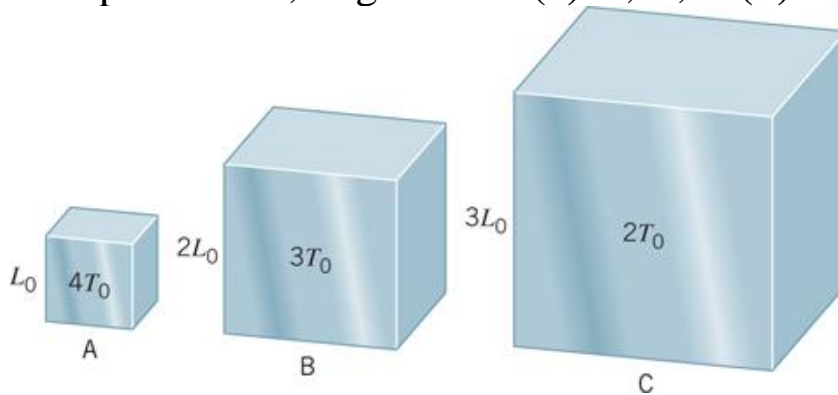
2. (b) This arrangement conducts more heat for two reasons. First, the temperature difference ΔT between the ends of each bar is greater in A than in B. Second, the cross-sectional area available for heat conduction is twice as large in A as in B. A greater cross-sectional area means more heat is conducted, everything else remaining the same.

$$Q = \frac{(kA\Delta T)t}{L},$$

FOC 8

Section 13.3 Radiation

8. Three cubes are made from the same material. As the drawing indicates, they have different sizes and temperatures. Rank the cubes according to the radiant energy they emit per second, largest first. **(a)** A, B, C **(b)** A, C, B **(c)** B, A, C **(d)** B, C, A **(e)** C, B, A




8. (c) The radiant energy emitted per second is given by $Q/t = e\sigma T^4 A$ (Equation 13.2). Note that it depends on the product of T^4 and the surface area A of the cube. The product $T^4 A$ is equal to $1944T_0^4 L_0^2$, $1536T_0^4 L_0^2$, and $864T_0^4 L_0^2$ for B, A, and C, respectively.

FOC 11

11. The emissivity e of object B is $\frac{1}{16}$ that of object A, although both objects are identical in size and shape. If the objects radiate the same energy per second, what is the ratio T_B/T_A of their Kelvin temperatures? **(a)** $\frac{1}{16}$ **(b)** $\frac{1}{4}$ **(c)** $\frac{1}{2}$ **(d)** 2 **(e)** 4

11. (d) The radiant energy emitted per second is given by $Q/t = e\sigma T^4 A$ (Equation 13.2), and it depends on the product eT^4 . Since the energy emitted per second is the same for both objects, the product eT^4 is the same for both. Since the emissivity of B is 16 times smaller than the emissivity of A, the temperature of B must be $\sqrt[4]{16} = 2$ times greater than A.

Pr. 10

10.  A wall in a house contains a single window. The window consists of a single pane of glass whose area is 0.16 m^2 and whose thickness is 2.0 mm . Treat the wall as a slab of the insulating material Styrofoam whose area and thickness are 18 m^2 and 0.10 m , respectively. Heat is lost via conduction through the wall and the window. The temperature difference between the inside and outside is the same for the wall and the window. Of the total heat lost by the wall and the window, what is the percentage lost by the window?

10. **REASONING** The heat lost by conduction through the wall is Q_{wall} and that lost through the window is Q_{window} . The total heat lost through the wall and window is $Q_{\text{wall}} + Q_{\text{window}}$. The percentage of the total heat lost by the window is

$$\text{Percentage} = \left(\frac{Q_{\text{window}}}{Q_{\text{wall}} + Q_{\text{window}}} \right) \times 100\% \quad (1)$$

The amount of heat Q conducted in a time t is given by

$$Q = \frac{(kA\Delta T)t}{L} \quad (13.1)$$

where k is the thermal conductivity, A is the area, ΔT is the temperature difference, and L is the thickness.

Pr. 10

Solution


SOLUTION Substituting Equation (13.1) into Equation (1), and letting the symbols “S” denote the Styrofoam wall and “G” the glass window, we have that

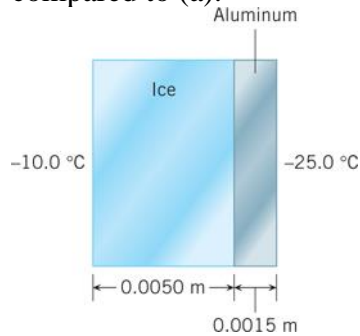
$$\begin{aligned}\text{Percentage} &= \left(\frac{Q_{\text{window}}}{Q_{\text{wall}} + Q_{\text{window}}} \right) \times 100\% \\ &= \left[\frac{\frac{k_G A_G (\Delta T) t}{L_G}}{\frac{k_S A_S (\Delta T) t}{L_S} + \frac{k_G A_G (\Delta T) t}{L_G}} \right] \times 100\% = \left(\frac{\frac{k_G A_G}{L_G}}{\frac{k_S A_S}{L_S} + \frac{k_G A_G}{L_G}} \right) \times 100\%\end{aligned}$$

Here we algebraically eliminated the temperature difference ΔT and the time t , since they are the same in each term. According to Table 13.1 the thermal conductivity of glass is $k_G = 0.80 \text{ J}/(\text{s} \cdot \text{m} \cdot \text{C}^\circ)$, while the value for Styrofoam is $k_S = 0.010 \text{ J}/(\text{s} \cdot \text{m} \cdot \text{C}^\circ)$. The percentage of the total heat lost by the window is

$$\begin{aligned}\text{Percentage} &= \left(\frac{\frac{k_G A_G}{L_G}}{\frac{k_S A_S}{L_S} + \frac{k_G A_G}{L_G}} \right) \times 100\% \\ &= \left\{ \frac{\frac{[0.80 \text{ J}/(\text{s} \cdot \text{m} \cdot \text{C}^\circ)](0.16 \text{ m}^2)}{2.0 \times 10^{-3} \text{ m}}}{\frac{[0.010 \text{ J}/(\text{s} \cdot \text{m} \cdot \text{C}^\circ)](18 \text{ m}^2)}{0.10 \text{ m}} + \frac{[0.80 \text{ J}/(\text{s} \cdot \text{m} \cdot \text{C}^\circ)](0.16 \text{ m}^2)}{2.0 \times 10^{-3} \text{ m}}} \right\} \times 100\% =\end{aligned}$$

Pr. 12

- *12.  Review Conceptual Example 5 before attempting this problem. To illustrate the effect of ice on the aluminum cooling plate, consider the drawing shown here and the data that it contains. Ignore any limitations due to significant figures. (a) Calculate the heat per second per square meter that is conducted through the ice–aluminum combination. (b) Calculate the heat per second per square meter that would be conducted through the aluminum if the ice were not present. Notice how much larger the answer is in (b) as compared to (a).



2. **REASONING** The energy Q conducted through a layer of material (thickness L and surface area A) in a time t is $Q = \frac{(kA\Delta T)t}{L}$ (Equation 13.1), where ΔT is the temperature difference between the two surfaces of area A and k is the thermal conductivity of the material. The heat conducted per second per square meter of area is

$$\frac{Q}{At} = \frac{k\Delta T}{L}$$

Pr. 12

Solution

SOLUTION

a. Before Equation (1) can be applied to the ice-aluminum combination, the temperature T at the interface must be determined. We find the temperature at the interface by noting that the heat conducted through the ice must be equal to the heat conducted through the aluminum: $Q_{\text{ice}} = Q_{\text{aluminum}}$. Applying Equation 13.1 to this condition, we have

$$\left(\frac{kA\Delta Tt}{L}\right)_{\text{ice}} = \left(\frac{kA\Delta Tt}{L}\right)_{\text{aluminum}} \quad (2)$$

or

$$\frac{[2.2 \text{ J}/(\text{s} \cdot \text{m} \cdot \text{C}^\circ)]A[(-10.0^\circ\text{C}) - T]t}{0.0050 \text{ m}} = \frac{[240 \text{ J}/(\text{s} \cdot \text{m} \cdot \text{C}^\circ)]A[T - (-25.0^\circ\text{C})]t}{0.0015 \text{ m}}$$

The factors A and t can be eliminated algebraically, and the thermal conductivities are given in Table 13.1. Solving for T gives $T = -24.959^\circ\text{C}$ for the temperature at the interface.

Applying Equation (1) to the ice leads to

$$\left(\frac{Q}{At}\right)_{\text{ice}} = \frac{[2.2 \text{ J}/(\text{s} \cdot \text{m} \cdot \text{C}^\circ)][(-10.0^\circ\text{C}) - (-24.959^\circ\text{C})]}{0.0050 \text{ m}} = \boxed{6.58 \times 10^3 \text{ J}/(\text{s} \cdot \text{m}^2)}$$


Since heat is not building up in the materials, the rate of heat transfer per unit area is the same throughout the ice-aluminum combination. Thus, this must be the heat per second per square meter that is conducted through the ice-aluminum combination.

b. Applying Equation (1) to the aluminum in the absence of any ice gives:

$$\left(\frac{Q}{At}\right)_{\text{Al}} = \frac{[240 \text{ J}/(\text{s} \cdot \text{m} \cdot \text{C}^\circ)][(-10.0^\circ\text{C}) - (-25.0^\circ\text{C})]}{0.0015 \text{ m}} = \boxed{2.40 \times 10^6 \text{ J}/(\text{s} \cdot \text{m}^2)}$$

Pr. 20

Section 13.3 Radiation

20.  Light bulb 1 operates with a filament temperature of 2700 K, whereas light bulb 2 has a filament temperature of 2100 K. Both filaments have the same emissivity, and both bulbs radiate the same power. Find the ratio A_1/A_2 of the filament areas of the bulbs.

20. **REASONING** According to Equation 6.10b, power P is the change in energy Q divided by the time t during which the change occurs, or $P = Q/t$. The power radiated by a filament is given by the Stefan-Boltzmann law as

$$P = \frac{Q}{t} = e \sigma T^4 A \quad (13.2)$$

where e is the emissivity, σ is the Stefan-Boltzmann constant, T is the temperature (in kelvins), and A is the surface area. This expression will be used to find the ratio of the filament areas of the bulbs.

SOLUTION Solving Equation (13.2) for the area, we have

$$A = \frac{P}{e \sigma T^4}$$


Taking the ratio of the areas gives

$$\frac{A_1}{A_2} = \frac{\frac{P_1}{e_1 \sigma T_1^4}}{\frac{P_2}{e_2 \sigma T_2^4}}$$

Setting $e_2 = e_1$, and $P_2 = P_1$, we have that

$$\frac{A_1}{A_2} = \frac{\frac{P_1}{e_1 \sigma T_1^4}}{\frac{P_1}{e_1 \sigma T_2^4}} = \frac{T_2^4}{T_1^4} = \frac{(2100 \text{ K})^4}{(2700 \text{ K})^4} = \boxed{0.37}$$

Pr. 25

25.  **SSM** A person eats a dessert that contains 260 Calories. (This “Calorie” unit, with a capital C, is the one used by nutritionists; 1 Calorie = 4186 J. See Section 12.7.) The skin temperature of this individual is 36 °C and that of her environment is 21 °C. The emissivity of her skin is 0.75 and its surface area is 1.3 m². How much time would it take for her to emit a *net* radiant energy from her body that is equal to the energy contained in this dessert?

25. **SSM** **REASONING** According to the discussion in Section 13.3, the net power P_{net} radiated by the person is $P_{\text{net}} = e\sigma A(T^4 - T_0^4)$, where e is the emissivity, σ is the Stefan-Boltzmann constant, A is the surface area, and T and T_0 are the temperatures of the person and the environment, respectively. Since power is the change in energy per unit time (see Equation 6.10b), the time t required for the person to emit the energy Q contained in the dessert is $t = Q/P_{\text{net}}$.

SOLUTION The time required to emit the energy from the dessert is



$$t = \frac{Q}{P_{\text{net}}} = \frac{Q}{e\sigma A(T^4 - T_0^4)}$$

The energy is $Q = (260 \text{ Calories})\left(\frac{4186 \text{ J}}{1 \text{ Calorie}}\right)$, and the Kelvin temperatures are

$T = 36 \text{ }^\circ\text{C} + 273 = 309 \text{ K}$ and $T_0 = 21 \text{ }^\circ\text{C} + 273 = 294 \text{ K}$. The time is

$$t = \frac{(260 \text{ Calories})\left(\frac{4186 \text{ J}}{1 \text{ Calorie}}\right)}{(0.75)\left[5.67 \times 10^{-8} \text{ J}/(\text{s} \cdot \text{m}^2 \cdot \text{K}^4)\right](1.3 \text{ m}^2)\left[(309 \text{ K})^4 - (294 \text{ K})^4\right]} = \boxed{1.2 \times 10^4 \text{ s}}$$

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*42.   A copper rod has a length of 1.5 m and a cross-sectional area of $4.0 \times 10^{-4} \text{ m}^2$. One end of the rod is in contact with boiling water and the other with a mixture of ice and water. What is the mass of ice per second that melts? Assume that no heat is lost through the side surface of the rod.

42. **REASONING** If m kilograms of ice melt in t seconds, then $Q = mL_f$ (Equation 12.5) joules of heat must be delivered to the ice through the copper rod in t seconds, where $L_f = 33.5 \times 10^4 \text{ J/kg}$ is the latent heat of fusion of water. The mass of ice per second that melts, then, is given by the ratio $\frac{m}{t}$. The rate $\frac{Q}{t}$ of heat flow through the copper rod is found from $\frac{Q}{t} = \frac{kA\Delta T}{L}$ (Equation 13.1), where k is the thermal conductivity of copper, A and L are, respectively, the cross-sectional area and length of the rod, and $\Delta T = 100.0 \text{ C}^\circ$ is the difference in temperature between the boiling water and the ice-water mixture.

SOLUTION Solving $Q = mL_f$ (Equation 12.5) for m yields $m = \frac{Q}{L_f}$. Dividing this by the elapsed time t , we obtain an expression for the mass of ice per second that melts:

$$\frac{m}{t} = \frac{\left(\frac{Q}{t}\right)}{L_f} \quad (1)$$

Substituting $\frac{Q}{t} = \frac{kA\Delta T}{L}$ (Equation 13.1) into Equation (1), we find that

$$\frac{m}{t} = \frac{kA\Delta T}{L_f L} = \frac{[390 \text{ J}/(\text{s} \cdot \text{m} \cdot \text{C}^\circ)](4.0 \times 10^{-4} \text{ m}^2)(100.0 \text{ C}^\circ)}{(33.5 \times 10^4 \text{ J/kg})(1.5 \text{ m})} = \boxed{3.1 \times 10^{-5} \text{ kg/s}}$$